α-CLUSTERING AND ABSOLUTE α-DECAY WIDTHS IN SPHERICAL NUCLEI

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The absolute α-decay in ^{212}Po is calculated using a shell-model description of the α-particle formation. It is found that high-lying shell-model configurations greatly enhance both the α-clustering features and the calculated α-decay width. The interaction among the nucleons that form the α-particle is included through correlated two-particle states.

One important feature of the shell model is that it provides a very convenient representation to solve the many-body Schrödinger equation. That is, of all the infinite single-particle shell-model states needed to completely span the Hilbert space (including the continuum) only very few are necessary to describe bound states in spherical nuclei. Yet, as soon as the number of active particles becomes large enough the number of shell-model configurations formed by those few single-particle states becomes unmanageably large. A similar difficulty appears when one wants to describe very high excited states. In particular, the clustering of nucleons on the nuclear surface requires the inclusion of very high single-particle states and in the case of α-decay even the continuum part of the single-particle spectrum has to be taken into account. High-lying configurations are important to describe processes that take place in the nuclear surface region because these configurations extend further out in the nuclear surface [14]. This is the reason why configuration mixing was found to have a tremendous importance in α-decay calculations [1–6]. In ref. [7] it was found that even the clustering of the pair of particles induced by the pairing interaction (giving rise to the pairing vibration state) proceeds through high-lying configurations. The α-decay widths are also strongly enhanced by the same effect [3,7]. Moreover, the inclusion of

high-lying configurations gives the proper tail of the α-cluster wavefunction and the α-width may be written in the classical form as [3]

$$\lambda_L = 2P_L(r_c)\, \gamma_L^2(r_c), \qquad (1)$$

where γ_L is the α-particle formation amplitude. If the shell-model wavefunction is assumed for the mother nucleus with the daughter nucleus as the core one obtains

$$\gamma_L(r_c) = (\hbar^2 r_c/2\mu)^{1/2} \int d\hat{r}_\alpha \int d\xi_\alpha \{\Phi_\alpha(\xi_\alpha)\, Y_L(r_\alpha)\}_1$$

$$\times \Psi_\alpha(r_1, r_2, r_3, r_4), \qquad (2)$$

where $\{\xi_\alpha\} = \{\xi_1, \xi_2, \xi_3\}$ are the internal coordinates of the α-particle. The intrinsic radial α-particle wavefunction is given by

$$\Phi_\alpha(\xi_\alpha) = \sqrt{\tfrac{1}{8}}(\nu_\alpha/\pi)^{9/4} \exp[\nu_\alpha(\xi_1^2 + \xi_2^2 + 2\xi_3^2)/4]\chi_\alpha, \quad (3)$$

We took the parameter entering eq. (3) as in ref. [3]. The shell-model wavefunction Ψ_α in (2) describes the motion of the four particles outside the core. In addition, only neutron–neutron (proton–proton) total spin $S_\nu = 0$ ($S_\pi = 0$) is assumed in eq. (2). The coordinates r_i ($i = 1, ..., 4$) fix the position of the four particles with respect to the core center. Finally r_α is the α-particle center-of-mass coordinate. The function P_L in eq. (1) is the Coulomb penetration factor and r_c is the radius beyond which only the Coulomb field is important. If the wavefunction of the mother nucleus

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is well described at large distance (where the nucleus density is very low) one can match this function to the irregular Coulomb function G_L, which describes the relative motion of the α-particle and the daughter nucleus. That is, in this region the Pauli principle has practically no effect and the α-decay process can be viewed as a two-body problem. In the language of the resonating group method [8,9] one could say that in the low density region $\Omega = (1 - K)^{-1/2}$, where $1 - K$ is the overlap kernel, is practically the identity. Although in refs. [3,7] high-lying configurations produced a large enhancement of α-decay width, only neutron—neutron and proton—proton interactions were included. Thus, only clusters of "dineutrons" and "diprotons" were taken into account in the α-particle wavefunction but protons and neutrons were completely uncorrelated from each other [7]. As a result, even when including many configurations the α-decay width in refs. [3,7] was too small compared with the experimental value by an order of magnitude. In these references the ^{212}Po wavefunction was written as

$$|^{212}\text{Po(gs)}\rangle = |^{210}\text{Po(gs)} \ ^{210}\text{Pb(gs)}\rangle , \qquad (4)$$

which, even without using high-lying configurations, describes the bound state properties of ^{212}Po [10] well. A proper description of the α-cluster would require either the inclusion (at least in principle) of all the ^{210}Pb and ^{210}Po states i.e.

$$\sum_{\alpha_2 \beta_2} X(\alpha_2\beta_2)|^{210}\text{Pb}(\alpha_2) \ ^{210}\text{Po}(\beta_2)\rangle$$

as suggested in ref. [7], or the inclusion of correlated states in ^{210}Bi. If there is a state in ^{210}Bi which shows to be clustered in the nuclear surface then one would expect that neutrons and protons in the α-particle may also be clustered through the same mechanism. In this case eq. (4) becomes

$$|^{212}\text{Po(gs)}\rangle = A|^{210}\text{Po(gs)} \ ^{210}\text{Pb(gs)}\rangle$$

$$+ B|^{210}\text{Bi}(\lambda) \ ^{210}\text{Bi}(\lambda)\rangle . \qquad (5)$$

This wavefunction would be exactly the same as eq. (4) if the overlap

$$O_{12} = \langle^{210}\text{Po(gs)} \ ^{210}\text{Pb(gs)}|^{210}\text{Bi}(\lambda) \ ^{210}\text{Bi}(\lambda)\rangle \qquad (6)$$

would be one. In general, the wavefunction amplitudes

A and B can be calculated if both (6) and

$$O_{22} = \langle^{210}\text{Bi}(\lambda) \ ^{210}\text{Bi}(\lambda)|^{210}\text{Bi}(\lambda) \ ^{210}\text{Bi}(\lambda)\rangle \qquad (7)$$

are known. This can be done using, for example, the multistep shell-model method (MSM) [11]).

Using the shell-model representation that includes the high-lying single particle states of ref. [7], we looked for a state in ^{210}Bi with clustering features. Perhaps not surprising, we found that the state $|^{210}\text{Bi}(0_1^+)\rangle$ has these features (in fact even more pronounced than the corresponding states $|^{210}\text{Pb}(0_1^+)\rangle$ and $|^{210}\text{Po}(0_1^+)\rangle$). This state was not found experimentally, but we assigned to it an excitation energy of 5 MeV, which corresponds to about the energy of the energy gap between major shell-model shells. With the single-particle states of ref. [7] and a neutron—proton pairing interaction we calculated the $|^{210}\text{Bi}(0_1^+)\rangle$ wave function. We found $O_{22} = 1.08$ and $O_{12} = 0.09$ which shows that the two basis states in eq. (5) are practically independent of each other. Once one knows these overlaps and the two-particle energies, the wave function (5) can be calculated [11]. Using the MSM we obtained $A = 0.98$, $B = 0.10$ and the ^{212}Po(gs) energy was reproduced within 100 keV. These values are the same as those that one would obtain using the standard shell-model calculation. Yet, these values of A and B may be considered as a first approximation to the corresponding values that would be obtained by a proper treatment of the neutron—proton correlations that induce the clustering of neutrons and protons to form the α-particle outside the surface of the mother nucleus.

The calculation of the α-width requires the computation of a nine-dimensional integral since the integral over r_α gives a factor 4π. We did this using the code D01FCF [12] which took about 11 h of CPU time in a VAX 11/78 computer.

The first thing we did was to check whether the α-particle was indeed clustered in the nuclear surface. This calculation was relatively easy since no integral was involved. As a function of different coordinates we plotted the square of the wave function (5). In fig. 1 we show one of such graphs. This figure shows quite clearly that our treatment gives a good description of the clustering of the α-particle at the nuclear surface.

We then calculated the α-decay half-lives and found that our results are practically independent of the distances between the daughter nucleus and the α-par-
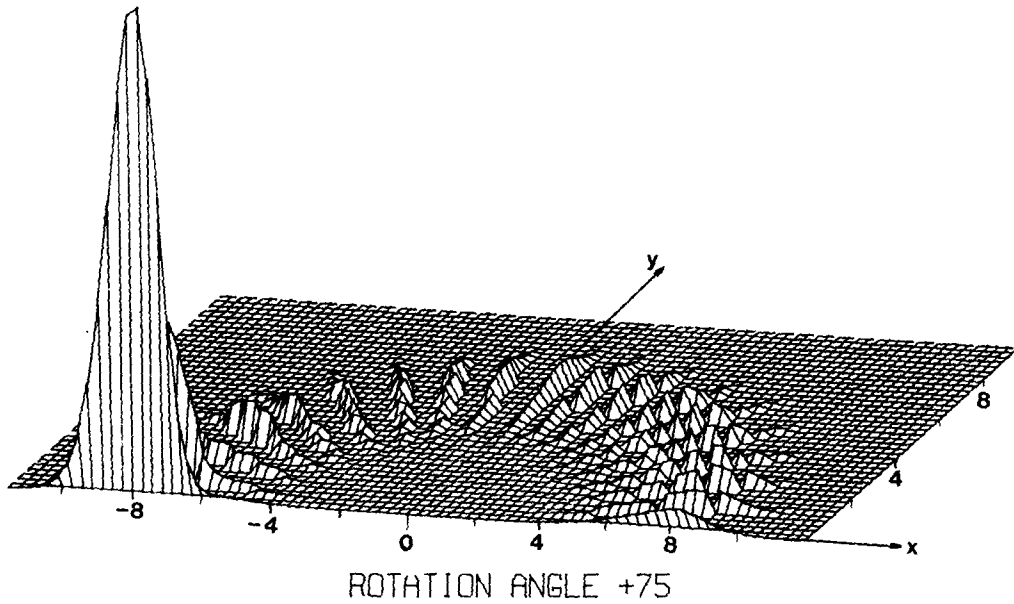
ROTATION ANGLE +75

Fig. 1. Square of the wave function of the α-particle moving around the ^{208}Pb core [eq. (5)]. Of the 12 spatial coordinates needed to describe this wave function, we chose all azimuthal angles $\phi_i = 0$, besides we chose $r_1 = r_2 = r_p$ and $r_3 = r_4 = r_n$, where 1 and 2 (3 and 4) label protons (neutrons), r_p (r_n) is the proton–proton (neutron–neutron) center-of-mass radius and the proton–proton (neutron–neutron) relative angle θ_p (θ_n) is $\theta_p = \theta_n = 0$. In this figure we located the two protons over the x-axis at a distance $x = -8$ fm. The two neutrons were allowed to move over the plane. Thus, this figure shows the effect of the neutron–proton interaction upon the α-particle wave function. With $B = 0$ the clustering seen around $x = -8$ fm disappears completely and one obtains an arc of large values of the wave function around the centre of coordinate with radius $r = 8$ fm (i.e. the small wrinkles seen in this figure become as important as the large peak). The rotation angle is the view angle. Distances are in fm.

ticle around the nuclear surface as shown in table 1.

However, the calculated ratio of half-lives at $r = 8.7$ fm is $R = T_{1/2}(\text{theor})/T_{1/2}(\text{exp}) = 31$. In ref. [3] the calculation of α-decay width without np-correla-

tion was performed for the same nucleus and the results were reported to be closer to experiment than ours. But in ref. [3] the single-particle wave functions were assumed to be the harmonic oscillator wave functions for which the Moshinski transformation is valid [13]. It is worthwhile to note that the value $B \approx 0.1$ in eq. (5) corresponds to the bound state of the nucleus ^{212}Po. The effective value of this admixture may be much more important in our case. In table 2

Table 1
Ratio $R = T_{1/2}(\text{th})/T_{1/2}(\text{exp})$ as a function of the distance (in fm) between the daughter nucleus and the α-particle. $T_{1/2}(\text{exp})$ was taken from ref. [14].

r(fm)	R
7.0	811
7.5	94
8.0	39
8.5	29
8.6	31
8.7	31
8.8	35
8.9	36
9.0	39
9.5	70
10.0	160

Table 2
Ratio R (see table 1) as a function of the neutron–proton mixing parameter B [eq. (5)] and for a distance $r = 8.7$ fm.

B	R
0.0	76
0.1	31
0.2	19.8
0.43	8.2
0.6	5.1
1.0	3.1

values of R as function of B are presented. We can see that by increasing B we can come quite close to the experimental results.

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