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ON ALPHA-DECAY AND ALPHA-CLUSTERING

Thesis

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This Thesis is presented for the assessment to the Election Board of the Scientific Committee of the Rudjer Bošković Institute with the aim of obtaining the PhD degree in Physics.

To my son Tvrtko Luka

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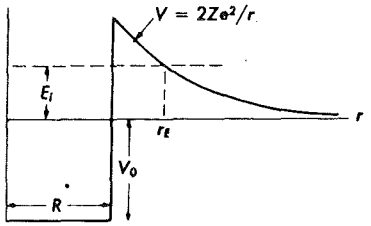
PROLOGUE

It is now exactly sixty years (1928-1988) since Gamow presented his pioneering paper on alpha-decay. In it, α–particle is supposed to exist preformed inside the nucleus, and it occurs in the outside world due to the tunnel effect. The description of the penetration of α through the Coulomb barrier of the nucleus was the first application of quantum mechanics to nuclear physics. Gamow basic idea is shown in the following illustration:



The decay constant λ (which equals $\ln 2/T_{1/2}$) is given by the product of penetration factor P and the reduced transition probability f:

$$\lambda = f\, P\ .$$

The f is the number of collisions per second with the barrier ("frequency factor"), and P is the fraction of collisions resulting in transmission:

$$P \approx\ exp\,(\frac{-2}{\hbar}\int_{R}^{2Ze^2/r_E}[2M_\alpha(\frac{2Ze^2}{r}-E_\alpha)]^{1/2}\,dr\,)$$

where M_α is the α–particle reduced mass, E_α is its energy, and the rest as in the figure. The integral is extended over the region in which the potential energy is larger than the kinetic energy of the α–particle, that is the region into which α–particle could not penetrate if classical mechanics were valid.

It was considered a great success of nuclear physics when Gamow was able to explain theoretically Geiger-Nuttal empirical law, connecting the decay constant with the energy of emitted alpha-particle. Gamow theory remained essentially unchanged for decades.

Only with the introduction of shell model, in early fifties, it was possible to formulate the first really microscopic theories and to calculate the reduced transition probability f inside the nucleus. Unfortunately enough, the discrepancies between theoretical and experimental absolute alpha-decay widths were disastrous: many orders of magnitude. Gradually it became clear that the continuum part of the single-particle spectrum was not properly treated. The continuum has a very important effect in the exit channel where alpha-particle is formed.

As a possible remedy, Resonating Group Method was proposed. In it, the partition into the final nucleus and the alpha-particle was naturally taken into account. The application of RGM to light nuclei was rather successful, but only in seventies it was applied to the alpha-decay in heavy nuclei where its success proved to be more than modest.

Although it has long been known that configuration mixing is important to calculate properly α–particle formation amplitude, only recently it was analysed into a great detail and applied to the case of α-decay of heavy nuclei.

1. INTRODUCTION

Before entering any discussion about α-decay I want to quote the following poem as an example of what one calls *the model* :

There are holes in the sky
where the rain gets in
but the holes are small:
that's why rain is thin.

Spike Milligan

Actually, we are more likely to think about sun than about rain in connection with α-particles. Simply because they are ^4He nuclei. And, as the name Helium indicates, they were first identified in the spectrum of the sun (in 1868).

But now ... let me start from the *very beginning* !

1.1 Nucleosynthesis

According to the big bang cosmological model, favoured at the present time, the universe is the result of an explosion from initial state with temperature $T > 10^{12}K$.
From that time it is expanding and cooling continuously, as in the following scenario (ref. 119):

1) 0.018 seconds after big bang. $T = 10^{11}K$.

The universe is filled with a mixture of matter and radiation in thermal equilibrium. The charge, baryon number and lepton number are equal or close to zero. There is one nucleon per 10^9 photons or electrons or neutrinos. Numbers of protons and neutrons are about the same. Processes:

$$\bar{\nu} + p \longrightarrow e^+ + \nu$$
$$\nu + n \longrightarrow e + p$$

take place.

2) 0.11 seconds later, $T = 3 \times 10^{10}K$.

With falling temperature it is easier for heavier neutrons to turn into lighter protons than vice versa. There are 38 % neutrons vs. 62 % protons.

3) 0.98 seconds later, $T = 10^{10}K$.

Proton-neutron balance is shifted to 24 % neutrons and 76 % protons.

4) 12.73 seconds later, $T = 3 \times 10^9K$.

Number of neutrons is still decreasing. There are 17 % neutrons and 83 % protons.

5) 2 minutes and 48.18 seconds later, $T = 10^9$.

"Deuterium bottleneck " prevents deuterium to hold together long enough to make possible building of heavier nuclei. The neutron-proton balance is now: 14 % neutrons and 86 % protons.

A little bit later, the temperature drops to the point at which deuterium nuclei hold together. All the remaining neutrons are built into ^4He. Just before nucleosynthesis started it was 13 % neutrons and 87 % protons.

As ^4He consists of two protons and two neutrons it means that the fraction by weight of helium was 26 % .

6) 31 minutes and 38 seconds after, $T = 3 \times 10^8 K$.

Nucleosynthesis have stopped. The nucleons exist either as free protons (hydrogen nuclei) or as ^4He (22-28 % by weight).

Further nucleosynthetic activity can not take place until the matter has condensed into stars massive enough to provide (from their gravitational energy) necessary high temperature to start nuclear reactions. From (ref. 1, 11) we learn the rest of the story about the nucleosynthesis of heavier elements.

It is certainly not the pure chance that α-particles are considered so important for nucleosynthesis. ^4He itself has some very unique features. It is the first completely saturated nucleus. As each nucleon has four internal degrees of freedom: spin up and down and isospin up and down, the wave-function in spin-isospin space can be fully antisymmetric only for four nucleons or less. To a good first approximation, α-particle spatial wave function is symmetric under the interchange of any two constituent nucleons (ref. 5).

From the very beginnings of nuclear physics it was known that α-particles emerge in the decay process of heavy nuclei (Maria Curie, 1900, E. Rutherford, 1908). The fact that α-particles take part in the creation of nucleus and they appear as a result of the radioactive decay of nucleus suggests the possibility that α-particle exists inside the nucleus.

1.2 Alpha-particle model. Clustering

Let me summarise:

1. α-particle is a second most abundant nucleus in the universe.
2. It is very stable and highly symmetric.
3. It plays an important role in synthesising of heavier nuclei.
4. It appears as a product of the decay process.

As a natural consequence comes the α-particle model of nucleus. During its long history (ref. 18) it evolved from the naive picture of point-like α-particles inside nuclear potential well, to the contemporary concepts of α-clustering.

Usually, the basic framework to discuss nuclear phenomena is the shell model. This model describes well properties of nuclei near closed shells, but it becomes increasingly complicated if more valence particles are present. In that cases collective model is applied. Apart from that, nuclei display a number of "intermediate" phenomena which are neither single-particle nor collective and we may call them clustering phenomena (ref. 70).

The extent and the forms of clustering are one of the most intriguing questions about nuclear structure. Alpha-clusters are of particular interest for obvious reasons of their high symmetry and stability (ref. 68). The calculations of α-clustering effects in nuclear matter (ref. 23) show that at normal densities of nuclear matter independent particle model is more stable, but at a density of one-third of the normal a transition from the plane wave to the α-cluster occurs. It means that α-clusters may be expected at the nuclear surface.

The existence of dynamical structures consisting of several nucleons (two-particle and four particle, α-like clusters) seems to be suggested by experiment. The piece of evidence for that is the energy of the cluster correlations separated from the total energy of nuclei from which the conclusion has been drawn that two-nucleon and four-nucleon clusters exist almost in any region of nuclei. The large variety of alpha-transfer experiments like $(^6Li, ^4He)$, $(^7Li, t)$ or $(^{16}O, ^{12}C)$ have also pointed out to such clusters. The reactions of the type $(p, ^4He)$ or $(^{12}C, ^{16}O)$ need preformed α-like structures on the nuclear surface (reaction time about 10^{-22} seconds) to be explained (ref. 110) .

Here one comment ought to be made. The α-particle reactions are usually described by means of the DWBA formalism. It implies the assumption that either incoming α-particle remains intact throughout the collision, or it is "absorbed" i.e. it disappears forever from the elastic channel. This seems to be quite simple-

minded approach. One should rather expect that the incoming α-particle becomes distorted (polarised). Moreover, after dissolution of incoming α-particle possibility for elastic scattering exists, etc. No matter how accurate our picture of transfer reactions is, from the commonly accepted two-nucleon clustering, the four-particle correlations should follow as well.

Often two pairs of nucleons with $T = 1$, $J = 0$ are called *quadruples*, and $T = 0$, $J = J_{max} = 2j$ are called *quartets*. Lacking a better word, we call the physical structures having SU4 symmetry *"quartets"*. They do seem to exist throughout the table of nuclides, but they can have quite different forms in different regions of it. At present our knowledge of the actual forms of the quartets is quite limited. I only list here some among conventional models, according to (ref. 42).

1) Wigner supermultiplets

The forces are assumed to be spin and isospin independent. SU4 is exact. To a certain degree these assumptions are fulfilled in the lightest nuclei.

2) Alpha-clustering

The four particles in α-quartet are close together and have strong interaction, while the interaction between the clusters is weak. The low average density of light nuclei allows the appearance of local saturated density regions separated by rather low density interstitial regions.

3) Aligned scheme, stretch state, HF wavefunction

Incomplete shell nuclei are filling in the intrinsic system first the minimum-m states (positive deformation) or the maximum-m states (negative deformation). Already without correlations these wave functions exhibit quartet structure in their interaction energies. From these uncorrelated wave functions correlated wave functions can be generated by configuration mixing.

4) Nilsson quartets

In a deformed basis the four particles allowed in a state of given $|K|$ form quite naturally quartets. The overlap between these particles is large and the interac-

tion with particles in other $|K|$ orbitals is substantially less. Again, correlations can arise by configuration mixing.

What can be the cause of the validity of the SU4 ?

The reason must be sought in the fact that the actual two-body forces have a short range, which is of the order of the Fermi wave length (= reciprocal Fermi momentum) in nuclei and that only four nucleons can come within one Fermi wavelength. This fact is independent of the existence (or non-existence) of correlations. It implies, however, that whatever the effective forces and the effective states, in the ground state only four particles at a time can interact strongly.

So, the reason for SU4 must be ascribed to the Pauli principle.

Of course, the mechanism of quarteting is yet far from being well understood. Some of the authors (ref. 70) suggest the following scheme for building of quartets :

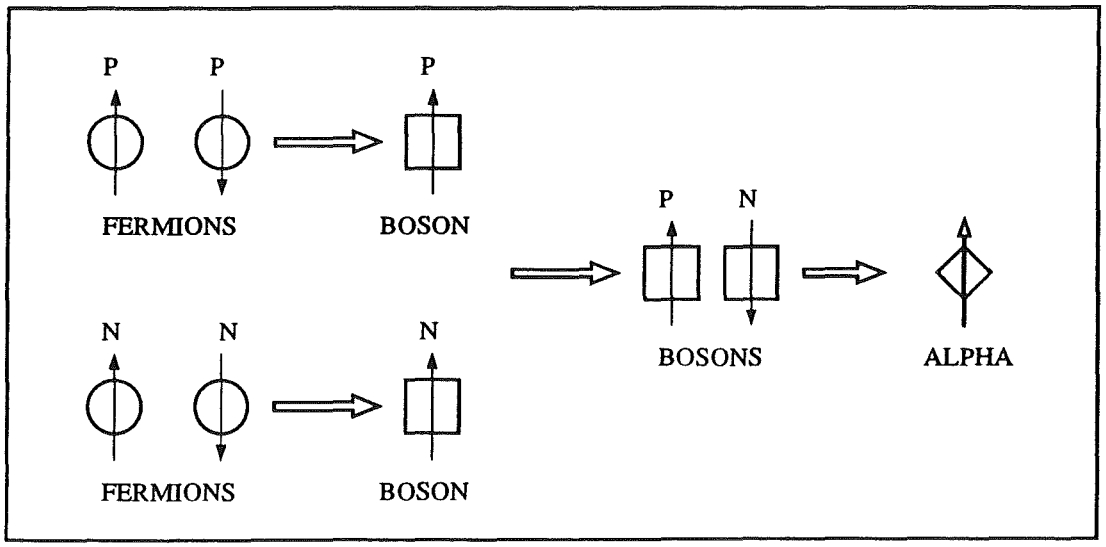


fig. 1.1

The starting point here is proton-proton and neutron-neutron pairing. Four-particle correlation produces "pairing of pairs" into alpha-clusters.

1.3 Odd-even staggering

One more evidence for four-particle correlations comes from odd-even-staggering of nuclear charge radii. Namely, the nuclear charge radii of odd-neutron isotopes are smaller than the averages of their even-neutron neighbours. With few exceptions, this behaviour is found over the whole table of nuclei. The explanation is the following.

The primary mechanism discriminating between even- and odd- neutron number is pairing. But there must be a mechanism which strongly couples the pairing properties of protons and neutrons. Otherwise the protons are hardly influenced by the fact that the neutron number is odd.

This mechanism must be universal and collective phenomenon, collectivity compensating for the smallness of the residual interaction. The only mechanism of that kind are four-body correlations or α-particle clustering (ref. 121, 122). Proton pairing increases the charge radius since it leads to partial occupation of levels above the Fermi energy at the cost of levels below, and the higher levels generally have larger rms radii. An odd neutron reduces neutrons support to proton pairing and that results in a reduction of the charge radius.

In (ref. 122) nuclear model systems with different four-body interactions are discussed with regard to four-body correlations and odd-even staggering of charge radii. Even with separable four-body force, the staggering can be reproduced almost quantitatively, the main contribution being due to one of the Hartree-Fock terms of this force.

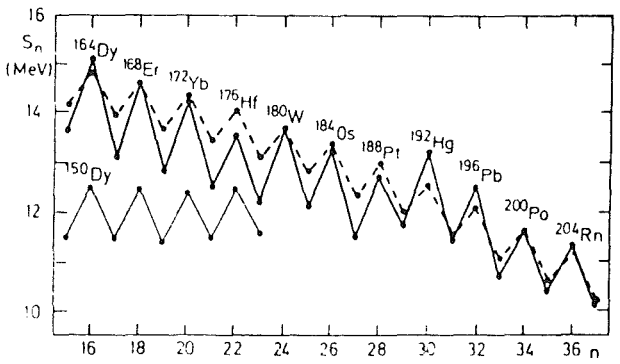


fig. 1.2 Odd-even staggering as a function of boson number

1.4 Four-body microscopic interpretation of the IBM

The Interacting Boson Model (IBM) of Arima and Iachello has been very successful in providing a phenomenological description of a wide variety of nuclear collective features. According to IBM, the fermion space is mapped onto a boson space by treating bosons as fermion pairs.
By comparison of boson Hamiltonian in the isospin s-boson space with fermion Hamiltonian we see that boson Hamiltonian consists of two parts : isoscalar part ($T = 0$) and isotensor part ($T = 2$), while the fermion Hamiltonian consists of only one, isovector term ($T = 1$). However, the boson isospin and fermion isospin are the same real isospin of the physical system. Hence, the boson and fermion Hamiltonians could not be considered as an image of one another.

It seems that there is only one way to bring the fermion Hamiltonian to the same isospin coupling as in the boson Hamiltonian, namely by an assumption that the two-body boson interaction has its image in the four-body effective fermion interaction. Such mapping exactly preserves the isospin coupling (ref. 112).

The new interpretation should answer the question why a four-body interaction term ought to be considered in the fermion-boson mapping. We know already that the IBM properly describes the physical systems which in fermion space can be described by a two-body interaction term alone. A possible answer is that the four-body fermion interaction coming from the two-body boson Hamiltonian can be considered as an effective four-body interaction following cut off of the fermion space of states. This may also partly explain why the IBM is so good in the interpretation of nuclear data.

1.5 α-decay and α-clustering

Inspired by the renewed interest in clustering phenomena (ref. 2-7, 13-16, 21, 23, 28, 30, 31, 40, 42, 46, 50, 54, 64, 67, 68, 79, 80, 84, 89, 92, 93, 97, 100-102, 110-113, 121-123) we tried to investigate clustering features of ^{212}Po as well as its α-decay half-life. Being made of ^{208}Pb core plus two protons and two neutrons ^{212}Po represents the ideal example of what one expects to be α-clustering in heavy nucleus. ^{208}Pb is well known as the best doubly magic nucleus, with $Z = 82$ and $N = 126$, thus representing a spherical inert core. It has been verified that it remains spherical even when many nucleons are added to the core. Therefore the core can be considered as the 0^+ vacuum state and its energy be set to zero. The properties of a nuclear states are derived from the valence nuclear configurations.

In (ref. 33, 34 and 35) it was shown that ^{212}Po described as ^{208}Pb inert core plus four valence nucleons exhibits a strong tendency towards formation of four-particle cluster in the surface region. Clustering itself is demonstrated to depend on the configuration mixing as well as on the proton-neutron interaction. In this picture the α-decay process follows Gamows two-step theory: first step being cluster formation, and the second step tunnelling through the potential barrier. The aim of present work is to show that the α-decay understood as a process of decaying of cluster state is in a good agreement with experimental half-lifes. The whole Introduction chapter was to put clustering of ^{212}Po into broader context of α-clustering and to suggest the relevance of the question of clustering features of nuclei.

2. ALPHA-DECAY ABSOLUTE REDUCED WIDTH AMPLITUDE

2.1 Description of the decay process

Natural alpha-decay is a process where the initial nucleus undergoes the following transformation :

$$^{Z}A_{N} \longrightarrow {}^{Z-2}(A-4)_{N-2} + \alpha.$$

We describe the initial nucleus B as consisting of the inert core A plus the cluster C outside of it :

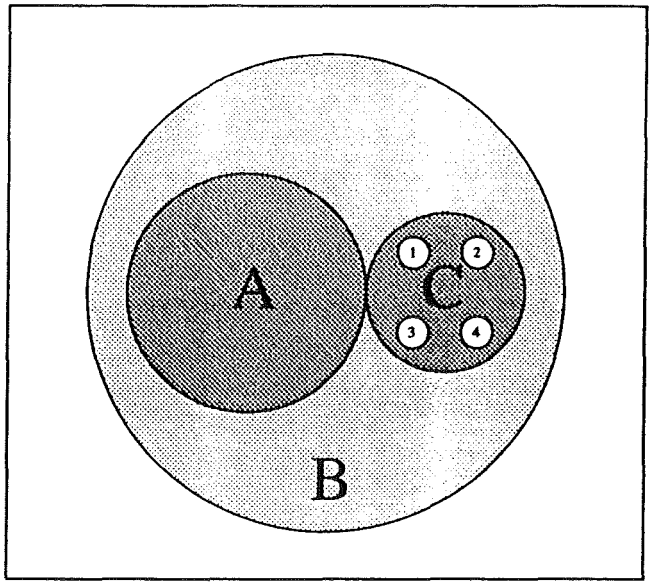


fig. 2.1 Initial state

According to (fig. 2.1) the wave function of the initial state may be written as

$$\Psi_i = [\Psi_{J_C M_C} \times \Psi_{J_A M_A}]_{J_B M_B} \tag{1}$$

or, if we explicitly write down the angular momentum couplings, it becomes:

$$\Psi_i = \sum_{M_C M_A} \langle J_C M_C J_A M_A | J_B M_B \rangle \; \Psi_{J_C M_C} \; \Psi_{J_A M_A}. \tag{2}$$

The core A is considered to be a mere spectator throughout the α-decay process. The final state Ψ_f is determined by the α-particle leaving a frozen core :

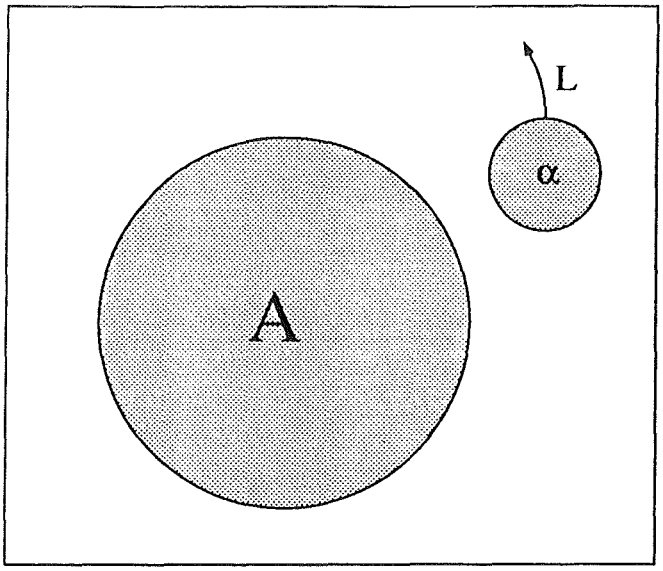


fig. 2.2 Final state

According to (fig. 2.2) the wave function of the final state may be written as

$$\Psi_f = [\Psi_{J_\alpha M_\alpha} \times Y_{LM} \times \Psi_{J_A M_A}]_{J_B M_B} \tag{3}$$

or, if we explicitly write down angular momentum couplings, it becomes

$$\Psi_f = \sum_\lambda \langle (LJ_\alpha)\lambda\ J_A; J_B | J_\alpha\ (J_A L)\lambda'; J_B \rangle [(Y_L \times \Psi_{J_\alpha})_\lambda \times \Psi_{J_A}]_{J_B}. \tag{4}$$

Now we take into account that $J_\alpha = 0$ and therefore we get the expression

$$\Psi_f = \sum_\lambda \langle (L0)L\ J_A; J_B | 0\ (J_A L)J_B; J_B \rangle [(Y_L \times \Psi_{J_{\alpha=0}})_L \times \Psi_{J_A}]_{J_B}. \tag{5}$$

Here we can use formula

$$\begin{Bmatrix} J_A\ L\ J_B \\ 0\ J_B\ L \end{Bmatrix} = \frac{(-1)^{J_A+L+J_B}}{\sqrt{(2J_B+1)(2L+1)}} \tag{6}$$

to obtain

$$\Psi_f = \frac{(-1)^{J_A+L+J_B}}{\sqrt{(2J_B+1)(2L+1)}} [(Y_L \times \Psi_{J_\alpha=0})_L \times \Psi_{J_A}]_{J_B}. \tag{7}$$

After writing down angular momentum couplings

$$\Psi_f = \frac{(-1)^{J_A+L+J_B}}{\sqrt{(2J_B+1)(2L+1)}} \sum_{M_L} \langle LM_L 00|LM_L\rangle \\ \times \sum_{M_L M_A} \langle LM_L J_A M_A|J_B M_B\rangle \; Y_{LM_L} \; \Psi_{J_a=0} \; \Psi_{J_a M_a}. \tag{8}$$

So the final state wave function has the following form

$$\Psi_f = \frac{(-1)^{J_A+L+J_B}}{\sqrt{(2J_B+1)/(2L+1)}} \\ \times \sum_{M_L M_A} \langle LM_L J_A M_A|J_B M_B\rangle \; Y_{LM_L} \; \Psi_{J_a=0} \; \Psi_{J_A M_A}. \tag{9}$$

The reduced alpha-decay width amplitude at given radius R is defined as the overlap integral between the initial and the final states

$$F(R) = \int \Psi_f^* \; \Psi_i \; d\tau \; . \tag{10}$$

If we make use of the definitions (2) and (9), the overlap integral becomes

$$F(R) = \sum_{M_A M_C} \langle J_C M_C J_A M_A|J_B M_B\rangle \int \Psi_{J_C M_C} \Psi_{J_A M_A} \\ \times \frac{(-1)^{J_A+L+J_B}}{\sqrt{(2J_B+1)/(2L+1)}} \sum_{M_A M_L} \langle LM_L J_A M_A|J_B M_B\rangle \; Y^*_{LM_L} \; \Psi^*_{J_a=0} \; \Psi^*_{J_A M_A} \; d\tau. \tag{11}$$

The core wave function is normalised according to

$$\int \Psi_A^*(\xi_A) \; \Psi_A(\xi_A) \; d\xi_A = 1 \tag{12}$$

and the integration may be carried out straightforward, so that the reduced width amplitude reads

$$F(R) = \sum_{M_A M_C M_L} \langle J_C M_C J_A M_A|J_B M_B\rangle \langle LM_L J_A M_A|J_B M_B\rangle \\ \times \frac{(-1)^{J_A+L+J_B}}{\sqrt{(2J_B+1)/(2L+1)}} \int \Psi_{J_C M_C} \; Y^*_{LM_L} \; \Psi^*_{J_{a=0}} \; d\tau. \tag{13}$$

The α-particle internal wave function is as usual of the Gaussian form

$$\Psi_{J_\alpha=0} = \frac{\left(2\beta^{3/2}\right)^{3/2}}{(1/2)!}\, e^{[-\beta/2(\xi_1^2+\xi_2^2+\xi_3^2)]}\,(4\pi)^{-3/2}\,\chi(1234) \tag{14}$$

where $\beta = 0.574 fm^{-2}$ is the extension parameter of the α-particle, $\chi(1234)$ is the spin function, while the coordinates are

$$\vec{\xi}_1 = \frac{1}{\sqrt{2}}\,(\vec{r}_1 - \vec{r}_2)$$

$$\vec{\xi}_2 = \frac{1}{\sqrt{2}}\,(\vec{r}_3 - \vec{r}_4) \tag{14.1}$$

$$\vec{\xi}_3 = \frac{1}{2}\,(\vec{r}_1 + \vec{r}_2 - \vec{r}_3 - \vec{r}_4).$$

The four-particle cluster wave function is built of all possible two-particle pairs. Therefore we distinguish two different contributions to cluster wave function, coming from (PP)-(NN) and (PN)-(PN) pairs. The cluster wave-function shown in (fig. 2.3) has the following form (for a moment we forget about four-particle wave function antisymmetrisation):

$$\Psi_{J_C M_C} \sim \sum_{\alpha_\nu,\alpha_\pi} X_{NNPP}(\alpha_\nu,\alpha_\pi)\Big\{\psi(\vec{r}_1\vec{r}_2;\alpha_\nu)\psi(\vec{r}_3\vec{r}_4;\alpha_\pi)\Big\}_{J_C} +$$

$$\sum_{\alpha_\delta,\alpha_{\delta'}} X_{NPNP}(\alpha_\delta,\alpha_{\delta'})\Big\{\psi(\vec{r}_1\vec{r}_3;\alpha_\delta)\psi(\vec{r}_2\vec{r}_4;\alpha_{\delta'}) - \psi(\vec{r}_1\vec{r}_4;\alpha_\delta)\psi(\vec{r}_2\vec{r}_3;\alpha_{\delta'})\Big\}_{J_C}. \tag{15}$$

The coefficients X_{NNPP} and X_{NPNP} are called the weights of (NN)-(PP) or (NP)-(NP) contribution, respectively. They are connected via the condition:

$$X_{NNPP}^2 + X_{NPNP}^2 = 1. \tag{16}$$

The two-particle wave functions are defined as

$$\psi(\vec{r}_1\vec{r}_2;\alpha_\nu) = \sum_{p\le q} X(pq;\alpha_\nu)\Big\{\varphi_p(\vec{r}_1)\varphi_q(\vec{r}_2)\Big\}_{\alpha_\nu}$$

$$\psi(\vec{r}_3\vec{r}_4;\alpha_\pi) = \sum_{r\le s} X(rs;\alpha_\pi)\Big\{\varphi_r(\vec{r}_3)\varphi_s(\vec{r}_4)\Big\}_{\alpha_\pi} \tag{17}$$

etc.

Let us go back to the overlap integral (13)

$$F(R) = \sum_{M_A M_C M_L} \langle J_C M_C J_A M_A | J_B M_B \rangle \langle L M_L J_A M_A | J_B M_B \rangle$$

$$\times \frac{(-1)^{J_A+L+J_B}}{\sqrt{(2J_B+1)/(2L+1)}} \int \Psi_{J_C M_C}\, Y^*_{LM_L}\, \Psi^*_{J_{\alpha}=0}\, d\tau.$$

For $J_A = 0$ (frozen core) and $L = 0$ that means $\Longrightarrow J_C = 0$ and $J_B = 0$. Consequently, we have

$$F(R) = \sum_{M_C} \langle J_C M_C 00 | J_B M_B \rangle \, \langle 0000 | J_B M_B \rangle \, \frac{(-1)^{J_B}}{\sqrt{(2J_B+1)}} \tag{18}$$

$$\times \int \Psi_{J_C=0}\, Y^*_{LM_L}\, \Psi^*_{J_{\alpha}=0}\, d\tau$$

so finally the expression for the overlap integral looks like as simple as

$$F(R) = \int \Psi_{J_C=0}\, Y^*_{LM_L} \Psi^*_{J_{\alpha}=0}\, d\tau. \tag{19}$$

Alpha-decay reduced width is defined via overlap integral as the product

$$\gamma_L(R) = \left(\frac{\hbar^2}{2\mu}\frac{R}{}\right)\; F(R) \tag{20}$$

where μ is α-particle reduced mass, and R is channel radius.

Alpha-decay width is given in a classical form

$$\Gamma_L(R) = 2\, \gamma_L^2(R)\, P_L(R) \tag{21.1}$$

and α-decay half-life is given by the equation

$$T_{1/2} = \frac{\hbar\, ln2}{\Gamma_L} = \frac{\hbar\, ln2}{2\, P_L\, \gamma_L^2}\,. \tag{21.2}$$

Up to this point the derivation is model independent. To make a concrete calculations, one has to decide about the single-particle radial wave functions. In our previous work we used Woods-Saxon radial wave functions. It implies the numerical integration of the 11-dimensional integral (19). To achieve the sufficient precision of integration routine means to consume enormous amunts of CPU-time on VAX11/780 computer. It is much more convenient to use H.O. wave functions as they make it possible to calculate overlap integral analytically.

2.2 Two-particle wave functions

As already mentioned before, the cluster wave function Ψ_C is built of two-particle wave functions (two-neutron, two-proton, neutron-proton). Two-particle wave functions are antisymmetrised so that for $T = 1$ space-spin part is antisymmetric, and for $T = 0$ space-spin part is symmetric. Radial wave functions are of the harmonic-oscillator type. The Hamiltonian which describes the independent motion of two particles in a harmonic-oscillator potential can be separated in $(\vec{r}_1, \vec{r}_2)$ space as well as in the relative $(\vec{r})$ and centre-of-mass coordinate $(\vec{R})$ (ref. 8, 10, 23.1, 80.1, 80.2, 90.1, 103). Talmi showed that the oscillator is the only potential that can be separated in both coordinate systems.
To exploit the simplicity afforded by the oscillator functions it is necessary to change from jj to LS coupling.

$$|jj\rangle = \sum_{LS} \langle L\ S|jj\rangle\ |L\ S\rangle \tag{22.1}$$

$$\langle L\ S|jj\rangle = \sum_{LS} \sqrt{(2j_1+1)(2j_2+1)(2L+1)(2S+1)} \begin{Bmatrix} l_1 & \frac{1}{2} & j_1 \\ l_2 & \frac{1}{2} & j_2 \\ L & S & J \end{Bmatrix} . \tag{22.2}$$

The antisymmetrized two-particle wave function is thereafter

$$\begin{aligned} [\varphi_{j_p l_p}(\vec{r}_1) \times \varphi_{j_q l_q}(\vec{r}_2)]_{JM} = {} & \frac{1}{\sqrt{2(1+\delta_{j_p j_q}\delta_{l_p l_q}\delta_{n_p n_q})}} \sum_{LS} \langle L\ S|j\ j\rangle \\ & \times \Big\{ \Big([\varphi_{n_p l_p}(\vec{r}_1) \times \varphi_{n_q l_q}(\vec{r}_2)]_{\lambda\mu} \times \chi_S(1,2) \Big)_{JM} \\ & + (-1)^T \Big([\varphi_{n_p l_p}(\vec{r}_2) \times \varphi_{n_q l_q}(\vec{r}_1)]_L \times \chi_S(2,1) \Big)_{JM} \Big\} \end{aligned} \tag{23}$$

where the single-particle wave functions are products of a harmonic oscillator radial wave function and a normalized spherical harmonic

$$\varphi_p(\vec{r}) = R_{n_p l_p}(br)\ Y_{l_p m_p}(\hat{r}) \tag{24.1}$$

b is harmonic oscillator strength ($b = 0.161$ for ^{208}Pb). A two-particle spin function is

$$\chi_{SM}(1,2) = \sum_{m,m'} \langle \frac{1}{2}\ \frac{1}{2}\ m\ m'|S\ M\rangle\ \chi_m(1)\ \chi_{m'}(2). \tag{24.2}$$

Transformation to the centre-of-mass and relative coordinate of two particles (Moshinsky -Talmi transformation) looks like (ref. 80.1)

$$[\varphi_{n_p l_p}(\vec{r}_1) \times \varphi_{n_q l_q}(\vec{r}_2)]_{\lambda\mu} = \sum_{nlNL} \langle nlNL;\lambda | n_p l_p n_q l_q;\lambda\rangle [\,\varphi_{nl}(\vec{r}) \times \varphi_{NL}(\vec{R})]_{\lambda\mu}. \quad (25)$$

Thus, if we Moshinsky-Talmi transform two-particle wave function (24) using relations

$$\chi_{SM}(2,1) = (-1)^{1+S}\chi_{SM}(1,2) \quad (26.1)$$

$$[\varphi_{n_p l_p}(\vec{r}_2) \times \varphi_{n_q l_q}(\vec{r}_1)] = (-1)^{l_p+l_q-L}[\varphi_{n_p l_p}(\vec{r}_1) \times \varphi_{n_q l_q}(\vec{r}_2)] \quad (26.2)$$

$$\langle nlNL;\lambda | n_p l_p n_q l_q;\lambda\rangle = (-1)^{L+\lambda}\langle nlNL;\lambda | n_q l_q n_p l_p;\lambda\rangle \quad (26.3)$$

$$(-1)^{l_p+l_q-L} = (-1)^{l} \quad (26.4)$$

$$2n_p + l_p + 2n_q + l_q = 2n + l + 2N + L \quad (26.5)$$

$$(-1)^{l_p+l_q} = (-1)^{L+l} \quad (26.6)$$

we obtain for the two-particle wave function, Moshinsky-Talmi transformed, and anti-symmetrised

$$[\varphi_{j_p l_p}(\vec{r}_1) \times \varphi_{j_q l_q}(\vec{r}_2)]_{JM} = \frac{1}{\sqrt{2(1+\delta_{j_p j_q}\delta_{l_p l_q}\delta_{n_p n_q})}} \sum_{\lambda S} \langle \lambda\ S | jj\rangle \times$$

$$\sum_{nlNL} \left\{1 - (-1)^{S+T+l}\right\} \left(\langle nlNL;\lambda | n_p l_p n_q l_q;\lambda\rangle [\varphi_{nl}(\vec{r}) \times \varphi_{NL}(\vec{R})]_\lambda \times \chi_S(1,2)\right)_{JM}. \quad (27)$$

These two-particle wave functions are the building blocks which we use to construct the four-particle cluster. We have four different kinds of two-particle wave functions: NN (spin-singlet), PP (spin-singlet), NP(spin-singlet) and NP (spin-triplet).

2.3 Question of spin

The spin state of two spin 1/2 particle system is either singlet or triplet.

The two-particle spin-singlet wave function is expressed via the single-particle ones

$$\chi_{00}(i,j) = \sum_{m_i,m_j} \langle \frac{1}{2} m_i \frac{1}{2} m_j | 0\,0 \rangle \; \chi_{\frac{1}{2}m_i}(i) \; \chi_{\frac{1}{2}m_j}(j) \tag{28}$$

or, explicitly

$$\chi_{00}(i,j) = \langle \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{-1}{2} | 0\,0 \rangle \; \chi_{\frac{1}{2}\frac{1}{2}}(i) \; \chi_{\frac{1}{2}\frac{-1}{2}}(j) + \langle \frac{1}{2} \frac{-1}{2} \frac{1}{2} \frac{1}{2} | 0\,0 \rangle \; \chi_{\frac{1}{2}\frac{-1}{2}}(i) \; \chi_{\frac{1}{2}\frac{1}{2}}(j). \tag{29}$$

We use the following formula for C-G coefficients

$$\langle lml - m | 00 \rangle = \frac{(-1)^{l-m}}{\sqrt{2l+1}} \tag{30}$$

to obtain

$$\chi_{00}(i,j) = \frac{1}{\sqrt{2}} [\chi_{\frac{1}{2}\frac{1}{2}}(i) \; \chi_{\frac{1}{2}\frac{-1}{2}}(j) - \chi_{\frac{1}{2}\frac{-1}{2}}(i) \; \chi_{\frac{1}{2}\frac{1}{2}}(j)]. \tag{31}$$

Taking into account that

$$\chi_{SM}(i,j) = (-1)^{1+S} \chi_{SM}(j,i) \tag{32}$$

we have

$$\chi_{00}(i,j) = \frac{2}{\sqrt{2}} \chi_{\frac{1}{2}\frac{1}{2}}(i) \; \chi_{\frac{1}{2}\frac{-1}{2}}(j) = \sqrt{2} \, \chi_{\frac{1}{2}\frac{1}{2}}(i) \; \chi_{\frac{1}{2}\frac{-1}{2}}(j). \tag{33}$$

The spin-triplet wave function may be expressed via single-particle ones as

$$\begin{aligned} \chi_{1-1}(i,j) &= \sum_{m_i m_j} \langle \frac{1}{2} m_i \frac{1}{2} m_j \, | \, 1 \; -1 \rangle \; \chi_{\frac{1}{2}m_i}(i) \; \chi_{\frac{1}{2}m_j}(j) \\ \chi_{1-1}(i,j) &= \langle \frac{1}{2} \frac{-1}{2} \frac{1}{2} \frac{-1}{2} | \, 1 \; -1 \rangle \; \chi_{\frac{1}{2}\frac{-1}{2}}(i) \; \chi_{\frac{1}{2}\frac{-1}{2}}(j) \end{aligned} \tag{34}$$

and making use of

$$\langle \frac{1}{2} \frac{-1}{2} \frac{1}{2} \frac{-1}{2} | \, 1 \; -1 \rangle = 1 \tag{35}$$

we obtain

$$\chi_{1-1}(i,j) = \chi_{\frac{1}{2}\frac{-1}{2}}(i)\ \chi_{\frac{1}{2}\frac{-1}{2}}(j). \tag{36}$$

Similarly, for the other spin projections

$$\chi_{11}(i,j) = \sum_{m_i m_j} \langle \frac{1}{2} m_i \frac{1}{2} m_j \,|\, 1\,1 \rangle\ \chi_{\frac{1}{2}m_i}(i)\ \chi_{\frac{1}{2}m_j}(j) \tag{37}$$

$$\chi_{11}(i,j) = \langle \frac{1}{2}\ \frac{1}{2}\ \frac{1}{2}\ \frac{1}{2} \,|\, 1\,1 \rangle\ \chi_{\frac{1}{2}\frac{1}{2}}(i)\ \chi_{\frac{1}{2}\frac{1}{2}}(j)$$

$$\langle \frac{1}{2}\ \frac{1}{2}\ \frac{1}{2}\ \frac{1}{2} | 1\,1 \rangle = 1 \tag{37.1}$$

$$\chi_{11}(i,j) = \chi_{\frac{1}{2}\frac{1}{2}}(i)\chi_{\frac{1}{2}\frac{1}{2}}(j) \tag{37.2}$$

and

$$\chi_{10}(i,j) = \sum_{m_i m_j} \langle \frac{1}{2} m_i \frac{1}{2} m_j | 10 \rangle \chi_{\frac{1}{2}m_i}(i) \chi_{\frac{1}{2}m_j}(j) \tag{38}$$

$$\langle \frac{1}{2}\ \frac{1}{2}\ \frac{1}{2}\ \frac{-1}{2} \,|\, 1\,0 \rangle = \langle \frac{1}{2}\ \frac{-1}{2}\ \frac{1}{2}\ \frac{1}{2} \,|\, 1\,0 \rangle = \frac{1}{\sqrt{2}} \tag{38.1}$$

$$\chi_{10}(i,j) = \frac{1}{\sqrt{2}} [\chi_{\frac{1}{2}\frac{1}{2}}(i)\ \chi_{\frac{1}{2}\frac{-1}{2}}(j) + \chi_{\frac{1}{2}\frac{-1}{2}}(i)\ \chi_{\frac{1}{2}\frac{1}{2}}(j)]. \tag{38.2}$$

Because of the Pauli principle NN and PP pairs are always in spin singlet state:

$$\chi_{00}(P,P) = \sqrt{2}\ \chi_{\frac{1}{2}\frac{1}{2}}(P)\ \chi_{\frac{1}{2}\frac{-1}{2}}(P) \tag{39.1}$$

$$\chi_{00}(N,N) = \sqrt{2}\ \chi_{\frac{1}{2}\frac{1}{2}}(N)\ \chi_{\frac{1}{2}\frac{-1}{2}}(N). \tag{39.2}$$

In much the same way, the NP system can be in one of the following spin states

$$\chi(N,P) = \begin{cases} \chi_{\frac{1}{2}\frac{1}{2}}(N)\ \chi_{\frac{1}{2}\frac{1}{2}}(P) \\ \chi_{\frac{1}{2}\frac{-1}{2}}(N)\ \chi_{\frac{1}{2}\frac{-1}{2}}(P) \\ \chi_{\frac{1}{2}\frac{1}{2}}(N)\ \chi_{\frac{1}{2}\frac{-1}{2}}(P) \\ \chi_{\frac{1}{2}\frac{-1}{2}}(N)\ \chi\frac{1}{2}\frac{1}{2}(P). \end{cases} \tag{40}$$

The spin functions corresponding to a triplet state of NP pair are

$$\chi_{11}(N,P) = \chi_{\frac{1}{2}\frac{1}{2}}(N)\ \chi_{\frac{1}{2}\frac{1}{2}}(P)$$

$$\chi_{10}(N,P) = \frac{1}{\sqrt{2}}\left[\chi_{\frac{1}{2}\frac{1}{2}}(N)\ \chi_{\frac{1}{2}\frac{-1}{2}}(P) + \chi_{\frac{1}{2}\frac{-1}{2}}(N)\ \chi_{\frac{1}{2}\frac{1}{2}}(P)\right] \tag{41}$$

$$\chi_{1-1}(N,P) = \chi_{\frac{1}{2}\frac{-1}{2}}(N)\ \chi_{\frac{1}{2}\frac{-1}{2}}(P).$$

The spin function corresponding to a singlet state of NP pair is

$$\chi_{00}(N,P) = \frac{1}{\sqrt{2}}\left[\chi_{\frac{1}{2}\frac{1}{2}}(N)\ \chi_{\frac{1}{2}\frac{-1}{2}}(P) - \chi_{\frac{1}{2}\frac{-1}{2}}(N)\ \chi_{\frac{1}{2}\frac{1}{2}}(P)\right]. \tag{42}$$

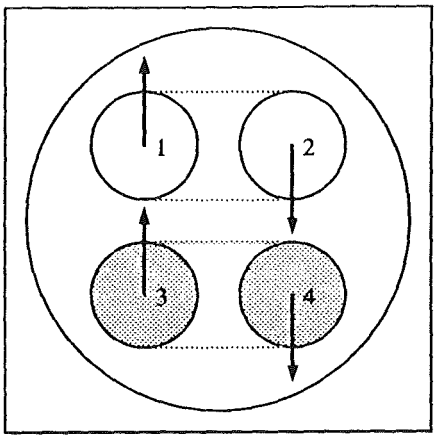
From the formulae above it follows that

$$\chi_{\frac{1}{2}\frac{1}{2}}(N)\ \chi_{\frac{1}{2}\frac{-1}{2}}(P) = \frac{1}{\sqrt{2}}\left[\chi_{10}(N,P) + \chi_{00}(N,P)\right]$$

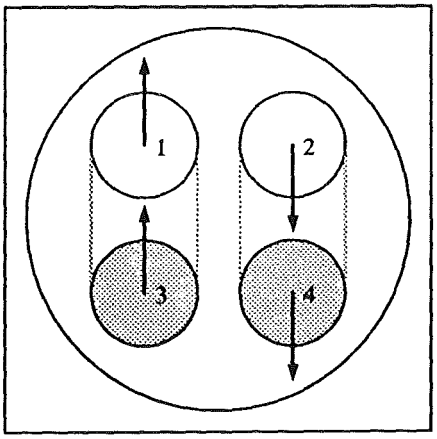
$$\chi_{\frac{1}{2}\frac{-1}{2}}(N)\ \chi_{\frac{1}{2}\frac{1}{2}}(P) = \frac{1}{\sqrt{2}}\left[\chi_{10}(N,P) - \chi_{00}(N,P)\right]. \tag{43}$$

Now we have all necessary instruments for studying the spin structure of four-particle cluster.

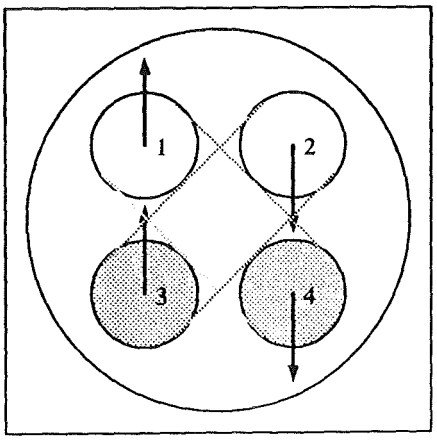
Possible two-particle spin functions



$$\chi_{00}(1,2)\,\chi_{00}(3,4) =$$
$$2\,[\chi_{\frac{1}{2}\frac{1}{2}}(1)\,\chi_{\frac{1}{2}\frac{-1}{2}}(2)]\,[\chi_{\frac{1}{2}\frac{1}{2}}(3)\,\chi_{\frac{1}{2}\frac{-1}{2}}(4)]$$



$$[\chi_{\frac{1}{2}\frac{1}{2}}(1)\,\chi_{\frac{1}{2}\frac{1}{2}}(3)]\,[\chi_{\frac{1}{2}\frac{-1}{2}}(2)\,\chi_{\frac{1}{2}\frac{-1}{2}}(4)]+$$
$$[\chi_{\frac{1}{2}\frac{-1}{2}}(1)\,\chi_{\frac{1}{2}\frac{-1}{2}}(3)][\chi_{\frac{1}{2}\frac{1}{2}}(2)\,\chi_{\frac{1}{2}\frac{-1}{2}}(4)] =$$
$$\chi_{11}(1,3)\,\chi_{1-1}(2,4) + \chi_{1-1}(1,3)\,\chi_{11}(2,4)$$



$$\chi_{\frac{1}{2}\frac{1}{2}}(N)\,\chi_{\frac{1}{2}\frac{-1}{2}}(P) = \frac{1}{\sqrt{2}}\,[\chi_{10}(N,P) + \chi_{00}(N,P)]$$
$$\chi_{\frac{1}{2}\frac{-1}{2}}(N)\,\chi_{\frac{1}{2}\frac{1}{2}}(P) = \frac{1}{\sqrt{2}}\,[\chi_{10}(N,P) - \chi_{00}(N,P)] \Rightarrow$$
$$[\chi_{\frac{1}{2}\frac{1}{2}}(1)\,\chi_{\frac{1}{2}\frac{-1}{2}}(4)][\chi_{\frac{1}{2}\frac{-1}{2}}(2)\,\chi_{\frac{1}{2}\frac{1}{2}}(3)]+$$
$$[\chi_{\frac{1}{2}\frac{-1}{2}}(1)\,\chi_{\frac{1}{2}\frac{1}{2}}(4)][\chi_{\frac{1}{2}\frac{1}{2}}(2)\,\chi_{\frac{1}{2}\frac{-1}{2}}(3)] =$$
$$\frac{1}{2}\,[\chi_{10}(1,4) + \chi_{00}(1,4)]\,[\chi_{10}(2,3) - \chi_{00}(2,3)]+$$
$$\frac{1}{2}\,[\chi_{10}(1,4) - \chi_{00}(1,4)]\,[\chi_{10}(2,3) + \chi_{00}(2,3)] =$$

$$\chi_{10}(1,4)\,\chi_{10}(2,3) - \chi_{00}(1,4)\,\chi_{00}(2,3)$$

fig. 2.3 Different spin couplings

2.4 Cluster wave function

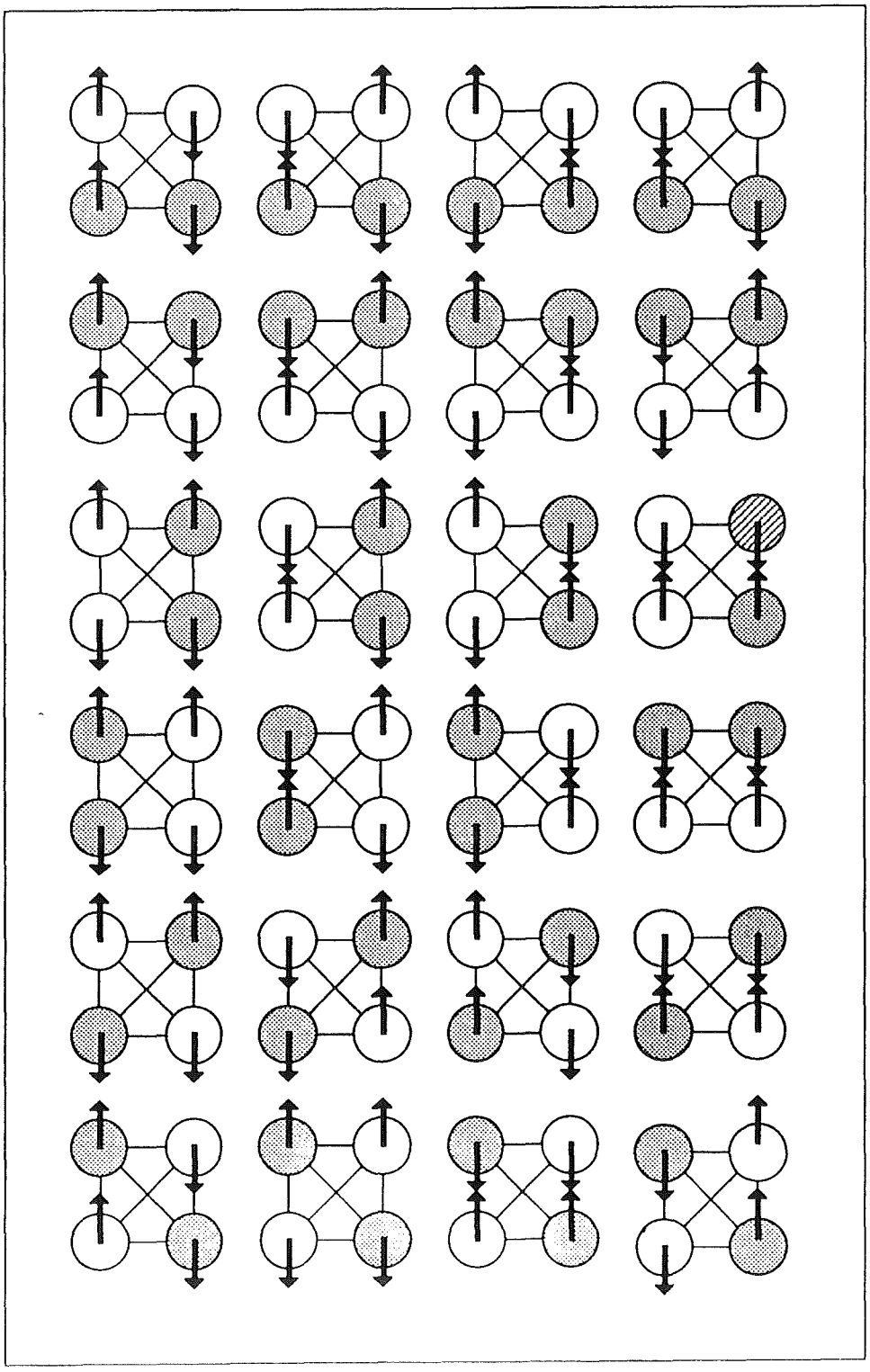
The antisymmetrized four-particle wave function may be written in a form of Slater determinant as:

$$\Psi(1234) = \frac{1}{\sqrt{4!}} \begin{vmatrix} \varphi_{11} & \varphi_{12} & \varphi_{13} & \varphi_{14} \\ \varphi_{21} & \varphi_{22} & \varphi_{23} & \varphi_{24} \\ \varphi_{31} & \varphi_{32} & \varphi_{33} & \varphi_{34} \\ \varphi_{41} & \varphi_{42} & \varphi_{43} & \varphi_{44} \end{vmatrix} \tag{44}$$

where φ_{32} denotes particle 3 on the radial position 2. After writing down the above determinant explicitly, one has

$$\begin{aligned} \Psi(1234) = \frac{1}{\sqrt{24}} (&\varphi_{11}\varphi_{22}\varphi_{33}\varphi_{44} + \varphi_{11}\varphi_{23}\varphi_{34}\varphi_{42} + \varphi_{11}\varphi_{24}\varphi_{32}\varphi_{43} - \\ &\varphi_{11}\varphi_{24}\varphi_{33}\varphi_{42} - \varphi_{11}\varphi_{23}\varphi_{32}\varphi_{44} - \varphi_{11}\varphi_{22}\varphi_{34}\varphi_{43} - \\ &\varphi_{12}\varphi_{21}\varphi_{33}\varphi_{44} - \varphi_{12}\varphi_{23}\varphi_{34}\varphi_{41} - \varphi_{12}\varphi_{24}\varphi_{31}\varphi_{43} + \\ &\varphi_{12}\varphi_{24}\varphi_{33}\varphi_{41} + \varphi_{12}\varphi_{23}\varphi_{31}\varphi_{44} + \varphi_{12}\varphi_{21}\varphi_{34}\varphi_{43} + \\ &\varphi_{13}\varphi_{21}\varphi_{32}\varphi_{44} + \varphi_{13}\varphi_{22}\varphi_{34}\varphi_{41} + \varphi_{13}\varphi_{24}\varphi_{31}\varphi_{42} - \\ &\varphi_{13}\varphi_{24}\varphi_{32}\varphi_{41} - \varphi_{13}\varphi_{22}\varphi_{31}\varphi_{44} - \varphi_{13}\varphi_{21}\varphi_{34}\varphi_{42} - \\ &\varphi_{14}\varphi_{21}\varphi_{32}\varphi_{43} - \varphi_{14}\varphi_{22}\varphi_{33}\varphi_{41} - \varphi_{14}\varphi_{23}\varphi_{31}\varphi_{42} + \\ &\varphi_{14}\varphi_{23}\varphi_{32}\varphi_{41} + \varphi_{14}\varphi_{22}\varphi_{31}\varphi_{43} + \varphi_{14}\varphi_{21}\varphi_{33}\varphi_{42}). \end{aligned} \tag{44.1}$$

fig. 2.4 The possible four-particle configurations



From (fig. 2.4) it is clear that there are six different four-particle configurations which are all equivalent to (fig. 2.3). Therefore, the four-particle cluster wave function may be written as

$$\Psi_{J_C}(\vec{r}_1\vec{r}_2\vec{r}_3\vec{r}_4) \sim \sum_{\alpha_\nu,\alpha_\pi} X_{NNPP}(\alpha_\nu,\alpha_\pi)[\psi(\vec{r}_1\vec{r}_2;\alpha_\nu)\psi(\vec{r}_3\vec{r}_4;\alpha_\pi)]_{J_C} +$$

$$\sum_{\alpha_\delta,\alpha_{\delta'}} X_{NPNP}(\alpha_\delta,\alpha_{\delta'})[\psi(\vec{r}_1\vec{r}_3;\alpha_\delta)\psi(\vec{r}_2\vec{r}_4;\alpha_{\delta'}) - \psi(\vec{r}_1\vec{r}_4;\alpha_\delta)\psi(\vec{r}_2\vec{r}_3;\alpha_{\delta'})]_{J_C}. \tag{45}$$

Expressing two-particle wave functions via single-particle ones, brings that formula to

$$\begin{aligned}\Psi_{J_C}(\vec{r}_1\vec{r}_2\vec{r}_3\vec{r}_4) \sim & \sum_{\alpha_\nu\alpha_\pi} X_{NNPP}(\alpha_\nu,\alpha_\pi) \\ & \times [\sum_{p\le q} X(pq;\alpha_\nu)\{\varphi_p(\vec{r}_1)\varphi_q(\vec{r}_2)\} \sum_{r\le s} X(rs;\alpha_\pi)\{\varphi_r(\vec{r}_3)\varphi_s(\vec{r}_4)\}]_{J_C} \\ & + \sum_{\alpha_\delta,\alpha_{\delta'}} X_{NPNP}(\alpha_\delta,\alpha_{\delta'}) \\ & \times [\sum_{p\le r} X(pr;\alpha_\delta)\{\varphi_p(\vec{r}_1)\varphi_r(\vec{r}_3)\} \sum_{q\le s} X(qs;\alpha_{\delta'})\{\varphi_q(\vec{r}_2)\varphi_s(\vec{r}_4)\}]_{J_C} \\ & - [\sum_{p\le s} X(ps;\alpha_\delta)\{\varphi_p(\vec{r}_1)\varphi_s(\vec{r}_4)\} \sum_{q\le r} X(qr;\alpha_{\delta'})\{\varphi_q(\vec{r}_2)\varphi_r(\vec{r}_3)\}]_{J_C}].\end{aligned} \tag{45.1}$$

2.5 (NN)–(PP) contribution

Let me concentrate for a moment on the first contribution to the cluster wave function, namely the one coming from (NN)-(PP) clusterisation. Using eq. (27) the cluster wave function becomes

$$\begin{aligned}
\Psi^{NNPP}_{J_C}(\vec{r}_1\vec{r}_2\vec{r}_3\vec{r}_4) = \Bigg\{ & \sum_{\alpha_\nu,\alpha_\pi} X_{NNPP}(\alpha_\nu,\alpha_\pi) \\
& \times \Bigg\{ \sum_{p\le q} X(pq;\alpha_\nu) \frac{1}{\sqrt{2(1+\delta_{j_pj_q}\delta_{l_pl_q}\delta_{n_pn_q})}} \sum_{\lambda_\nu S_\nu} \langle \lambda_\nu S_\nu | j_p j_q \rangle \\
& \times \sum_{n_\nu l_\nu N_\nu L_\nu} \langle n_\nu l_\nu N_\nu L_\nu; \lambda_\nu | n_p l_p n_q l_q; \lambda_\nu \rangle \\
& \times [1-(-1)^{S_\nu+T_\nu+l_\nu}][[\varphi_{n_\nu l_\nu}(\vec{r}_\nu) \times \varphi_{N_\nu L_\nu}(\vec{R}_\nu)]_{\lambda_\nu} \times \chi(1,2)]_{J_\nu M_\nu} \Bigg\} \\
& \times \Bigg\{ \sum_{r\le s} X(rs;\alpha_\pi) \frac{1}{\sqrt{2(1+\delta_{j_rj_s}\delta_{l_rl_s}\delta_{n_rn_s})}} \sum_{\lambda_\pi S_\pi} \langle \lambda_\pi S_\pi | j_r j_s \rangle \\
& \times \sum_{n_\pi l_\pi N_\pi L_\pi} \langle n_\pi l_\pi N_\pi L_\pi; \lambda_\pi | n_r l_r n_s l_s; \lambda_\pi \rangle [1-(-1)^{S_\pi+T_\pi+l_\pi}] \\
& \times [[\varphi_{n_\pi l_\pi}(\vec{r}_\pi) \times \varphi_{N_\pi L_\pi}(\vec{R}_\pi)]_{\lambda_\pi} \times \chi(3,4)]_{J_\pi M_\pi} \Bigg\} \Bigg\}_{J_C} .
\end{aligned} \tag{46}$$

In the above equation a coupling of the following type appears

$$\begin{aligned}
A = \Bigg\{ & [[\varphi_{n_\nu l_\nu}(\vec{r}_\nu) \times \varphi_{N_\nu L_\nu}(\vec{R}_\nu)]_{\lambda_\nu} \times \chi(1,2)]_{J_\nu M_\nu} \\
& \times [[\varphi_{n_\pi l_\pi}(\vec{r}_\pi) \times \varphi_{N_\pi L_\pi}(\vec{R}_\pi)]_{\lambda_\pi} \times \chi(3,4)]_{J_\pi M_\pi} \Bigg\}_{J_C}
\end{aligned} \tag{47}$$

the angular part of which may be written as

$$B = [[(Y_{l_\nu} Y_{L_\nu})_{\lambda_\nu}\ \chi^{S_\nu}]_{J_\nu} [(Y_{l_\pi} Y_{L_\pi})_{\lambda_\pi}\ \chi^{S_\pi}]_{J_\pi}]_{J_C}$$

$$= \sum_{J_C M_{J_C}} \langle (\lambda_\nu S_\nu) J_\nu\ (\lambda_\pi S_\pi) J_\pi; J_C | (\lambda_\nu \lambda_\pi) L_C\ (S_\nu S_\pi) S_C; J_C \rangle \tag{48}$$

$$\times \left\{ [(Y_{l_\nu} Y_{L_\nu})_{\lambda_\nu}\ (Y_{l_\pi}\ Y_{L_\pi})_{\lambda_\pi}]_{L_C}\ (\chi^{S_\nu}\ \chi^{S_\pi}) S_C \right\}_{J_C}.$$

First one recouples spherical harmonics

$$[(Y_{l_\nu} Y_{L_\nu})\lambda_\nu\ (Y_{l_\pi} Y_{L_\pi})\lambda_\pi]_{L_C} =$$

$$\sum_{L_C M_{L_C}} \langle (l_\nu L_\nu)\lambda_\nu\ (l_\pi L_\pi)\lambda_\pi; L_C | (l_\nu l_\pi) l_C\ (L_\nu L_\pi) L_C; L_C \rangle \tag{49}$$

$$\times [(Y_{l_\nu} Y_{l_\pi})_{l_c}\ (Y_{L_\nu} Y_{L_\pi})_{L_C}]_{L_C}$$

to obtain

$$B = \sum_{J_C M_{J_C}} \langle (\lambda_\nu S_\nu) J_\nu\ (\lambda_\pi S_\pi) J_\pi; J_C | (\lambda_\nu \lambda_\pi) L_C\ (S_\nu S_\pi) S_C; J_C \rangle$$

$$\times \langle L_C M_{L_C} S_C M_{S_C} | J_C M_{J_C} \rangle$$

$$\times \sum_{L_C M_{L_C}} \langle (l_\nu L_\nu)\lambda_\nu\ (l_\pi L_\pi)\lambda_\pi; L_C | (l_\nu l_\pi) l_C\ (L_\nu L_\pi) L_C; L_C \rangle \tag{50}$$

$$\times (\chi^{S_\nu} \chi^{S_\pi})_{S_C}\ [(Y_{l_\nu} Y_{l_\pi})_{l_c} \times (Y_{L_\nu} Y_{L_\pi})_{L_C}]_{L_C}.$$

Making the Moshinsky-Talmi transformation from $N_\nu L_\nu, N_\pi L_\pi$ pair centre-of-mass coordinates to $n_C l_C, N_C L_C$ cluster relative and centre-of-mass coordinates one obtains

$$R_{N_\nu L_\nu}\ R_{N_\pi L_\pi}\ (Y_{L_\nu} Y_{L_\pi})_{L_C} =$$

$$\sum_{n_C l_C N_C L_C} \langle n_C l_C N_C L_C; J_C | N_\nu L_\nu N_\pi L_\pi; J_C \rangle\ R_{n_C l_C}\ R_{N_C L_C}\ (Y_{l_C} Y_{L_C})_{L_C}. \tag{51}$$

If we use the condition $S_C = 0 \quad S_\pi = 0 \quad S_\nu = 0$ (PP and NN pair spin singlet state), we get

$$B = \sum_{J_C M_{J_C}} \langle (\lambda_\nu 0) J_\nu \ (\lambda_\pi 0) J_\pi; J_C | (\lambda_\nu \lambda_\pi) L_C \ (00) 0; J_C \rangle \langle L_C M_{L_C} 00 | J_C M_{J_C} \rangle$$

$$\times \sum_{L_C M_{L_C}} \langle ((l_\nu L_\nu) \lambda_\nu \ (l_\pi L_\pi) \lambda_\pi; L_C | (l_\nu l_\pi) l_C \ (L_\nu L_\pi) L_C; L_C \rangle$$

$$\times (\chi^{S_\nu = 0} \ \chi^{S_\pi = 0})_{S_C = 0} \ [(Y_{l_\nu} Y_{l_\pi})_{l_c} \times (Y_{L_\nu} Y_{L_\pi})_{L_C}]_{L_C} \tag{52}$$

or

$$B = \sum_{J_C M_{J_C}} \langle (\lambda_\nu 0) J_\nu \ (\lambda_\pi 0) J_\pi; J_C | (\lambda_\nu \lambda_\pi) L_C \ (00) 0; J_C \rangle \langle L_C M_{L_C} 00 | J_C M_{J_C} \rangle$$

$$\times \sum_{L_C M_{L_C}} \langle ((l_\nu L_\nu) \lambda_\nu \ (l_\pi L_\pi) \lambda_\pi; L_C | (l_\nu l_\pi) l_C \ (L_\nu L_\pi) L_C; L_C \rangle$$

$$\times (\chi^{S_\nu = 0} \chi^{S_\pi = 0})_{S_C = 0} \ \langle l_C m_{l_C} L_C M_{L_C} | L_C M_{L_C} \rangle (Y_{l_\nu} \ Y_{l_\pi})_{l_c} (Y_{L_\nu} \ Y_{L_\pi})_{L_C}. \tag{53}$$

The first 9-J symbol is

$$\langle (\lambda_\nu 0) J_\nu \ (\lambda_\pi 0) J_\pi; J_C | (\lambda_\nu \lambda_\pi) L_C \ (00) 0; J_C \rangle = \hat{J}_\nu \hat{J}_\pi \hat{L}_C \begin{Bmatrix} \lambda_\nu & 0 & J_\nu \\ \lambda_\pi & 0 & J_\pi \\ L_C & 0 & J_C \end{Bmatrix}$$

$$= \frac{(-1)^{\lambda_\nu + J_\pi + L_C}}{\hat{L}_C} \hat{J}_\nu \hat{J}_\pi \hat{L}_C \begin{Bmatrix} J_\nu \ \lambda_\nu \ 0 \\ \lambda_\pi J_\pi L_C \end{Bmatrix} \tag{53.1}$$

$$= (-1)^{\lambda_\nu + J_\pi + L_C} \ \hat{J}_\nu \hat{J}_\pi \ \frac{1}{\hat{\lambda}_\nu} \ \frac{1}{\hat{\lambda}_\pi} \ \delta(J_\nu, \lambda_\nu) \ \delta(J_\pi, \lambda_\pi)$$

$$= (-1)^{\lambda_\nu + \lambda_\pi + L_C} \ \delta(J_\nu, \lambda_\nu) \ \delta(J_\pi, \lambda_\pi).$$

Thus we have

$$B = \sum_{J_C M_{J_C}} (-1)^{\lambda_\nu+\lambda_\pi+L_C}\ \delta(J_\nu,\lambda_\nu)\ \delta(J_\pi,\lambda_\pi)\ \langle L_C M_{L_C} 00|J_C M_{J_C}\rangle$$

$$\times \sum_{L_C M_{L_C}} \langle (l_\nu L_\nu)\lambda_\nu\ (l_\pi L_\pi)\lambda_\pi; L_C|(l_\nu l_\pi) l_C\ (L_\nu L_\pi) L_C; L_C\rangle \qquad (54)$$

$$\times \langle l\ m_l L_C M_{L_C}|L_C M_{L_C}\rangle\ (Y_{l_\nu} Y_{l_\pi})_{l_c}\ (Y_{L_\nu} Y_{L_\pi})_{L_C} (\chi^{S_\nu=0}\chi^{S_\pi=0})_{S_C=0}.$$

The whole expression for A may be thereafter written as

$$A = \sum_{J_C M_{J_C}} (-1)^{\lambda_\nu+\lambda_\pi+L_C}\ \delta(J_\nu,\lambda_\nu)\ \delta(J_\pi,\lambda_\pi)\ \langle L_C M_{L_C} 00|J_C M_{J_C}\rangle$$

$$\times \sum_{L_C M_{L_C}} \langle (l_\nu L_\nu)\lambda_\nu\ (l_\pi L_\pi)\lambda_\pi; L_C|(l_\nu l_\pi) l_C\ (L_\nu L_\pi) L_C; L_C\rangle \qquad (55)$$

$$\times \langle l_C m_{l_C} L_C M_{L_C}|L_C M_{L_C}\rangle (Y_{l_\nu} Y_{l_\pi})_{l_c} (Y_{L_\nu} Y_{L_\pi})_{L_C}$$

$$\times (\chi^{S_\nu=0}\chi^{S_\pi=0})_{S_C=0}\ R_{n_\nu l_\nu}\ R_{n_\pi l_\pi}\ R_{N_\nu L_\nu}\ R_{N_\pi L_\pi}.$$

After Moshinsky-Talmi transformation $(N_\nu L_\nu; N_\pi L_\pi) \longrightarrow (n_C l_C; N_C L_C)$ it reads

$$A = \sum_{J_C M_{J_C}} (-1)^{\lambda_\nu+\lambda_\pi+L_C}\ \delta(J_\nu,\lambda_\nu)\ \delta(J_\pi,\lambda_\pi)\ \langle L_C M_{L_C} 00|J_C M_{J_C}\rangle$$

$$\times \sum_{L_C M_{L_C}} \langle (l_\nu L_\nu)\lambda_\nu\ (l_\pi L_\pi)\lambda_\pi; L_C|(l_\nu l_\pi) l_C\ (L_\nu L_\pi) L_C; L_C\rangle \qquad (56)$$

$$\times \langle l_C m_{l_C} L_C M_{L_C}|L_C M_{L_C}\rangle\ (Y_{l_\nu} Y_{l_\pi})_{l_c}\ R_{n_\nu l_\nu}\ R_{n_\pi l_\pi}\ (\chi^{S_\nu=0}\chi^{S_\pi=0})_{S_C=0}$$

$$\times \sum_{n_C l_C N_C L_C} \langle n_C l_C N_C L_C; J_C|N_\nu L_\nu N_\pi L_\pi; J_C\rangle\ (Y_{l_C} Y_{L_C})_{L_C}\ R_{n_C l_C}\ R_{N_C L_C}.$$

At this point we use the assumptions $J_A = 0$ $L_\alpha = 0 \Longrightarrow J_B = 0$, $J_C = 0$ and $(S_C = 0,\ S_\pi = 0,\ S_\nu = 0) \Longrightarrow L_C = 0$ to obtain

$$
\begin{aligned}
A = {} & (-1)^{\lambda_\nu + \lambda_\pi}\, \delta(J_\nu, \lambda_\nu)\, \delta(J_\pi, \lambda_\pi)\, \langle 0\,0\,0\,0|0\,0\rangle \\
& \times \langle (l_\nu L_\nu)\lambda_\nu\, (l_\pi L_\pi)\lambda_\pi; 0|(l_\nu l_\pi)0\, (L_\nu L_\pi)0; 0\rangle\ \ \delta(l_c 0) \\
& \times \langle 0\,0\,0\,0|0\,0\rangle\ (Y_{l_\nu} Y_{l_\pi})_0\ R_{n_\nu l_\nu}\ R_{n_\pi l_\pi} (\chi^{S_\nu = 0} \chi^{S_\pi = 0})_{S_C = 0} \\
& \times \sum_{n_C N_C} \langle n_C\, 0\, N_C\, 0; 0|N_\nu L_\nu N_\pi L_\pi; 0\rangle\ (Y_{l_C = 0}\ Y_{L_C = 0})_{L_C = 0}\ R_{n_C 0}\ R_{N_C 0}
\end{aligned}
\tag{57}
$$

and finally

$$
\begin{aligned}
A = {} & (-1)^{\lambda_\nu + \lambda_\pi}\ \langle (l_\nu \lambda_\nu)\lambda_\nu\ (l_\pi \lambda_\pi)\lambda_\pi; 0|(l_\nu l_\pi)0\ (L_\nu L_\pi)0; 0\rangle \\
& \times (Y_{l_\nu}(\hat{\mathbf{r}}_\nu)\ Y_{l_\pi}(\hat{\mathbf{r}}_\pi))_0\ (\chi^{S_\nu = 0} \chi^{S_\pi = 0})_{S_C = 0}\ R_{n_\nu l_\nu}(r_\nu)\ R_{n_\pi l_\pi}(r_\pi) \\
& \times \sum_{n_C N_C} \langle n_C\ 0\ N_C\ 0; 0|N_\nu \lambda_\nu N_\pi \lambda_\pi; 0\rangle \\
& \times\ (Y_{L_C = 0}(\hat{\mathbf{R}}_C)\ Y_{l_C = 0}(\hat{\mathbf{r}}_C))_0\ R_{n_C l_C}(r_C)\ R_{N_C L_C}(R_C).
\end{aligned}
\tag{58}
$$

Here

$$
(Y_{l_\nu}(\hat{\mathbf{r}}_\nu)\ Y_{l_\pi}(\hat{\mathbf{r}}_\pi))_0 = \langle l_\nu\ m_\nu\ l_\pi\ m_\pi|0\ 0\rangle\ Y_{l_\nu}(\hat{\mathbf{r}}_\nu)\ Y_{l_\pi}(\hat{\mathbf{r}}_\pi) \tag{59}
$$

$$
(Y_{L_C = 0}(\hat{\mathbf{R}}_C)\ Y_{l_C = 0}(\hat{\mathbf{r}}_C))_0 = \langle 0\ 0\ 0\ 0|0\ 0\rangle\ Y_{L_C = 0}(\hat{\mathbf{R}}_C)\ Y_{l_C = 0}(\hat{\mathbf{r}}_C). \tag{60}
$$

Now we can calculate the overlap integral (19)

$$F(R) = \int \Psi_{J_C=0}\, Y^*_{LM_L}\, \Psi^*_{J_a=0}\, d\tau$$

with the cluster wave function of the following form

$$\begin{aligned}
\Psi^{NNPP}_{J_C=0}(\vec{r}_1\vec{r}_2\vec{r}_3\vec{r}_4) = & \sum_{\alpha_\nu,\alpha_\pi} X_{NNPP}(\alpha_\nu,\alpha_\pi) \\
& \times \sum_{p\le q} \frac{[1-(-1)^{S_\nu+T_\nu+l_\nu}]X(pq;\alpha_\nu)}{\sqrt{2(1+\delta_{j_pj_q}\delta_{l_pl_q}\delta_{n_pn_q})}} \\
& \times \sum_{\lambda_\nu S_\nu=0} \langle \lambda_\nu S_\nu | j_p j_q\rangle \sum_{n_\nu N_\nu \lambda_\nu} \langle n_\nu l_\nu N_\nu \lambda_\nu; \lambda_\nu | n_p l_p n_q l_q; \lambda_\nu\rangle \\
& \times \sum_{r\le s} \frac{[1-(-1)^{S_\pi+T_\pi+l_\pi}]X(rs;\alpha_\pi)}{\sqrt{2(1+\delta_{j_rj_s}\delta_{l_rl_s}\delta_{n_rn_s})}} \\
& \times \sum_{\lambda_\pi S_\pi=0} \langle \lambda_\pi S_\pi | j_r j_s\rangle \sum_{n_\pi N_\pi \lambda_\pi} \langle n_\pi l_\pi N_\pi \lambda_\pi; \lambda_\pi | n_r l_r n_s l_s; \lambda_\pi\rangle \\
& \times (-1)^{\lambda_\nu+\lambda_\pi} \langle (l_\nu\lambda_\nu)\lambda_\nu (l_\pi\lambda_\pi)\lambda_\pi; 0 | (l_\nu l_\pi) 0 (L_\nu L_\pi) 0; 0\rangle \times \\
& \times \sum_{n_C N_C} \langle n_C 0 N_C 0; 0 | N_\nu \lambda_\nu N_\pi \lambda_\pi; 0\rangle \ \langle l_\nu m_\nu l_\pi m_\pi | 00\rangle \\
& \times Y_{l_\nu}(\hat{\mathbf{r}}_\nu)\ Y_{l_\pi}(\hat{\mathbf{r}}_\pi) (Y_{L_C=0}(\hat{\mathbf{R}}_C)\ Y_{l_C=0}(\hat{\mathbf{r}}_C))_0 (\chi^{S_\nu=0}\chi^{S_\pi=0})_{S_C=0} \\
& \times R_{n_\nu l_\nu}(r_\nu)\ R_{n_\pi l_\pi}(r_\pi)\ R_{n_C l_C}(r_C)\ R_{N_C L_C}(R_C).
\end{aligned} \tag{61}$$

As mentioned before, the integration is made analytically.

2.6 Angular integration

In order to calculate the overlap integral, one shall first perform the integration over angular coordinates. Using formulas

$$Y_{00} = \frac{1}{\sqrt{4\pi}} \tag{62.1}$$

$$\int Y^*_{LM}(\hat{\mathbf{R}})\, Y_{lm}(\hat{\mathbf{R}})\, d\hat{\mathbf{R}} = \delta(Ll)\, \delta(Mm) \tag{62.2}$$

we obtain

$$\int Y^*_{L=0}(\hat{\mathbf{R}}_C)\, Y_{L_C M_C}(\hat{\mathbf{R}}_C)\, d\hat{\mathbf{R}}_C = \delta(L_C 0)\, \delta(M_C 0) \tag{63.1}$$

$$\int Y_{l_C m_C}(\hat{\mathbf{r}}_C)\, d\hat{\mathbf{r}}_C = \frac{4\pi}{\sqrt{4\pi}}\, \delta(l_C 0)\, \delta(m_C 0) = \sqrt{4\pi}\, \delta(l_C 0)\, \delta(m_C 0) \tag{63.2}$$

$$\int Y_{l_\nu m_\nu}(\hat{\mathbf{r}}_\nu)\, d\hat{\mathbf{r}}_\nu = \sqrt{4\pi}\, \delta(l_\nu 0)\, \delta(m_\nu 0) \tag{63.3}$$

$$\int Y_{l_\pi m_\pi}(\hat{\mathbf{r}}_\pi)\, d\hat{\mathbf{r}}_\pi = \sqrt{4\pi}\, \delta(l_\pi 0)\, \delta(m_\pi 0). \tag{63.4}$$

All in all, the angular integrations result in the factor

$$(\sqrt{4\pi})^3\, \delta(L_C 0)\, \delta(l_C 0)\, \delta(l_\nu 0)\, \delta(l_\pi 0).$$

Therefore, what remains is to calculate the integral

$$F^{S_C=0}_{NNPP}(R_C) = (4\pi)^{3/2}\, \delta(L_C 0)\, \delta(l_C 0)\, \delta(l_\nu 0)\, \delta(l_\pi 0) \times \int_0^\infty \Psi_{J_C=0}\, \Psi^*_{J_\alpha=0}\, r_C^2\, dr_C\, r_\nu^2\, dr_\nu\, r_\pi^2\, dr_\pi \tag{64}$$

with $\Psi_{J_C=0}$ given by eq. (61) and $\Psi_{J_\alpha=0}$ given by eq. (14).

As we are now discussing (NN)-(PP) contribution, it means that the pair spins are $S_\nu = S_\pi = 0$, isospins are $T_\nu = T_\pi = 1$ and orbital angular momenta are $l_\nu = l_\pi = 0$ (the last is the condition coming from angular integration). Subsequently, the phase factor appearing in eq. (61) is

$$[1 - (-1)^{S_\nu + T_\nu + l_\nu}] = [1 - (-1)^{S_\pi + T_\pi + l_\pi}] = [1 - (-1)^{0+1+0}] = 2 \tag{64.1}$$

With the help of eqs. (64), (61), (14) and (64.1) the overlap integral becomes

$$F^{S_C=0}_{NNPP}(R_C) = \left(\frac{2\beta^{3/2}}{(1/2)!}\right)^{3/2} \int e^{-\frac{\beta}{2}[\xi_1^2+\xi_2^2+\xi_3^2]} \sum_{\alpha_\nu,\alpha_\pi} X_{NNPP}(\alpha_\nu,\alpha_\pi)$$

$$\times \Bigg\{ \sum_{p\le q} \frac{2X(pq;\alpha_\nu)}{\sqrt{2(1+\delta_{j_pj_q}\delta_{l_pl_q}\delta_{n_pn_q})}}$$

$$\times \sum_{\lambda_\nu S_\nu=0} \langle \lambda_\nu S_\nu | j_p j_q \rangle \sum_{n_\nu N_\nu \lambda_\nu} \langle n_\nu l_\nu N_\nu \lambda_\nu; \lambda_\nu | n_p l_p n_q l_q; \lambda_\nu \rangle$$

$$\times \sum_{r\le s} \frac{2X(rs;\alpha_\pi)}{\sqrt{2(1+\delta_{j_rj_s}\delta_{l_rl_s}\delta_{n_rn_s})}} \qquad (65)$$

$$\times \sum_{\lambda_\pi S_\pi=0} \langle \lambda_\pi S_\pi | j_r j_s \rangle \sum_{n_\pi N_\pi \lambda_\pi} \langle n_\pi l_\pi N_\pi \lambda_\pi; \lambda_\pi | n_r l_r n_s l_s; \lambda_\pi \rangle$$

$$\times \sum_{n_C N_C} \langle n_C 0 N_C 0; 0 | N_\nu \lambda_\nu N_\pi \lambda_\pi; 0 \rangle \langle l_\nu m_\nu l_\pi m_\pi | 00 \rangle$$

$$\times (-1)^{\lambda_\nu+\lambda_\pi} \langle (0\lambda_\nu)\lambda_\nu\ (0\lambda_\pi)\lambda_\pi; 0 | (00)0\ (00)0; 0 \rangle\ (\chi^{S_\nu=0}\chi^{S_\pi=0})_{S_C=0}$$

$$\times R_{n_\nu 0}(r_\nu)\ R_{n_\pi 0}(r_\pi)\ R_{n_C 0}(r_C)\ R_{N_C 0}(R_C)\ r_\nu^2\ dr_\nu\ r_\pi^2\ dr_\pi\ r_C^2\ dr_C \Bigg\}.$$

Making use of:

$$\langle (0\lambda_\nu)\lambda_\nu\ (0\lambda_\pi)\lambda_\pi; 0 | (00)0\ (00)0; 0 \rangle \implies \lambda_\pi = \lambda_\nu$$

$$\langle (0\lambda)\lambda\ (\lambda 0)\lambda; 0 | (00)0\ (00)0; 0 \rangle = 1 \qquad (66)$$

$$(-1)^{\lambda_\pi+\lambda_\nu} = 1$$

one finally obtains:

$$F^{S_C=0}_{NNPP}(R_C) = \left(\frac{2\beta^{3/2}}{(1/2)!}\right)^{3/2} \sum_{\alpha_\nu,\alpha_\pi} X_{NNPP}(\alpha_\nu,\alpha_\pi)$$

$$\times \sum_{p\le q} \frac{2X(pq;\alpha_\nu)}{\sqrt{2(1+\delta_{j_pj_q}\delta_{l_pl_q}\delta_{n_pn_q})}} \sum_{r\le s} \frac{2X(rs;\alpha_\pi)}{\sqrt{2(1+\delta_{j_rj_s}\delta_{l_rl_s}\delta_{n_rn_s})}}$$

$$\times \sum_{\lambda_\nu\lambda_\pi} \langle \lambda_\nu S_\nu = 0 | j_p j_q \rangle \langle \lambda_\pi S_\pi = 0 | j_r j_s \rangle$$

$$\times \sum_{n_\nu N_\nu} \langle n_\nu 0 N_\nu \lambda_\nu; \lambda_\nu | n_p l_p n_q l_q; \lambda_\nu \rangle \sum_{n_\pi N_\pi} \langle n_\pi 0 N_\pi \lambda_\pi; \lambda_\pi | n_r l_r n_s l_s; \lambda_\pi \rangle \qquad (67)$$

$$\times \sum_{n_C N_C} \langle n_C 0 N_C 0; 0 | N_\nu \lambda_\nu N_\pi \lambda_\pi; 0 \rangle \; (\chi^{S_\nu=0}\chi^{S_\pi=0})_{S_C=0}$$

$$\times \int_0^\infty e^{(-\frac{\beta}{2}\xi_1^2)} \, R_{n_\nu 0}(r_\nu) \, r_\nu^2 \, dr_\nu \int_0^\infty e^{(-\frac{\beta}{2}\xi_2^2)} \, R_{n_\pi 0}(r_\pi) \, r_\pi^2 \, dr_\pi$$

$$\times \int_0^\infty e^{(-\frac{\beta}{2}\xi_3^2)} \, R_{n_C 0}(r_C) \, r_C^2 \, dr_C \times R_{N_C 0}(4b; R_C).$$

The coordinates are defined in the following way:

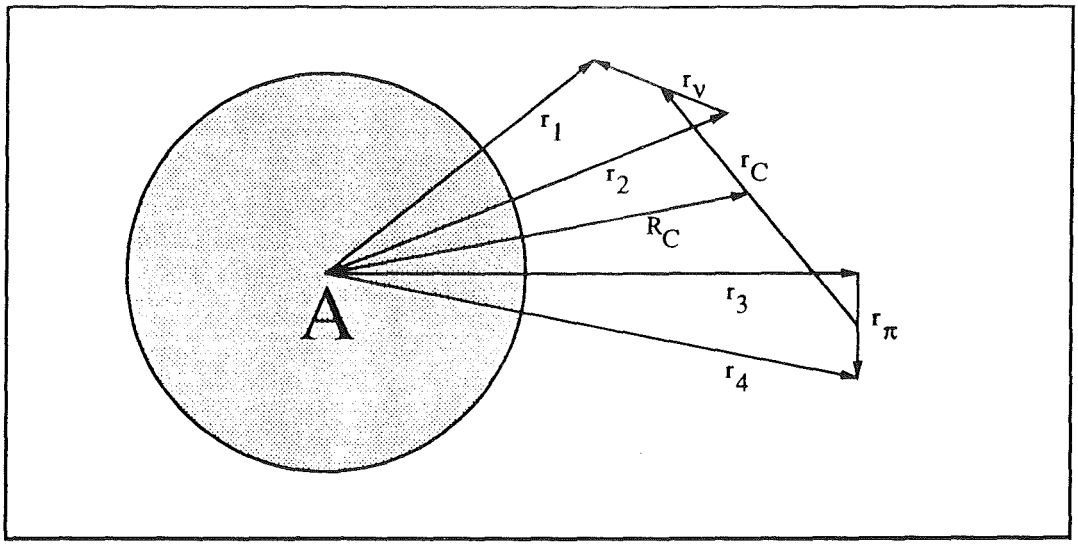


fig. 2.5 The coordinates

The definitions of $\vec{\xi}_1$,$\vec{\xi}_2$ and $\vec{\xi}_3$ are given by eq.(14.1), while

$$\vec{\xi}_4 = \frac{1}{2}(\vec{r}_1 + \vec{r}_2 + \vec{r}_3 + \vec{r}_4) \tag{68.1}$$

and

$$\vec{r}_\nu = \frac{1}{\sqrt{2}}(\vec{r}_1 - \vec{r}_2) \equiv \vec{\xi}_1$$

$$\vec{r}_\pi = \frac{1}{\sqrt{2}}(\vec{r}_3 - \vec{r}_4) \equiv \vec{\xi}_2$$

$$\vec{r}_C = \frac{1}{\sqrt{2}}(\vec{R}_\nu - \vec{R}_\pi) \equiv \vec{\xi}_3 \tag{68.2}$$

$$\vec{R}_\nu = \frac{1}{\sqrt{2}}(\vec{r}_1 + \vec{r}_2)$$

$$\vec{R}_\pi = \frac{1}{\sqrt{2}}(\vec{r}_3 + \vec{r}_4).$$

We use the invariance of an oscillator potential under the transformation

$$\vec{r}_1, \vec{r}_2, \vec{r}_3, \vec{r}_4 \longrightarrow \vec{\xi}_1, \vec{\xi}_2, \vec{\xi}_3, \vec{\xi}_4$$
$$\Longrightarrow \vec{r}_1^{\,2} + \vec{r}_2^{\,2} + \vec{r}_3^{\,2} + \vec{r}_4^{\,2} = \vec{\xi}_1^{\,2} + \vec{\xi}_2^{\,2} + \vec{\xi}_3^{\,2} + \vec{\xi}_4^{\,2}. \tag{69.1}$$

With the definitions of coordinates given above, we get

$$\vec{\xi}_1^{\,2} + \vec{\xi}_2^{\,2} + \vec{\xi}_3^{\,2} = \frac{1}{2}(\vec{r}_1 - \vec{r}_2)^2 + \frac{1}{2}(\vec{r}_3 - \vec{r}_4)^2 + \frac{1}{4}[(\vec{r}_1 + \vec{r}_2) - (\vec{r}_3 + \vec{r}_4)]^2$$

$$= \frac{1}{2}(\vec{r}_1 - \vec{r}_2)^2 + \frac{1}{2}(\vec{r}_3 - \vec{r}_4)^2 \tag{69.2}$$

$$+ \frac{1}{4}[(\vec{r}_1 + \vec{r}_2)^2 + (\vec{r}_3 + \vec{r}_4)^2 - 2(\vec{r}_1 + \vec{r}_2)(\vec{r}_3 + \vec{r}_4)]$$

$$\vec{\xi}_4^{\,2} = \frac{1}{4}[(\vec{r}_1 + \vec{r}_2) + (\vec{r}_3 + \vec{r}_4)]^2 =$$

$$= \frac{1}{4}[(\vec{r}_1 + \vec{r}_2)^2 + (\vec{r}_3 + \vec{r}_4)^2 + 2(\vec{r}_1 + \vec{r}_2)(\vec{r}_3 + \vec{r}_4)] \tag{69.3}$$

$$\vec{\xi}_1^{\,2} + \vec{\xi}_2^{\,2} + \vec{\xi}_3^{\,2} + \vec{\xi}_4^{\,2} = \frac{1}{2}[\vec{r}_1^{\,2} + \vec{r}_2^{\,2} - 2\vec{r}_1\vec{r}_2 + \vec{r}_3^{\,2} + \vec{r}_4^{\,2} - 2\vec{r}_3\vec{r}_4$$
$$+ \vec{r}_1^{\,2} + \vec{r}_2^{\,2} + 2\vec{r}_1\vec{r}_2 + \vec{r}_3^{\,2} + \vec{r}_4^{\,2} + 2\vec{r}_3\vec{r}_4] \tag{69.4}$$

$$\vec{\xi}_1^{\,2} + \vec{\xi}_2^{\,2} + \vec{\xi}_3^{\,2} + \vec{\xi}_4^{\,2} = \vec{r}_1^{\,2} + \vec{r}_2^{\,2} + \vec{r}_3^{\,2} + \vec{r}_4^{\,2} \tag{69.5}$$

$$\vec{R}_{CM} = \frac{1}{4}\left[\vec{r}_1 + \vec{r}_2 + \vec{r}_3 + \vec{r}_4\right] = \frac{1}{2}\vec{\xi}_4. \tag{69.6}$$

That means that we have the following relation (ref. 88.1):

$$\vec{r}_1^{\,2} + \vec{r}_2^{\,2} + \vec{r}_3^{\,2} + \vec{r}_4^{\,2} = \vec{\xi}_1^{\,2} + \vec{\xi}_2^{\,2} + \vec{\xi}_3^{\,2} + 4\vec{R}_C^{\,2} \tag{69.7}$$

or, in other words,

$$\vec{\xi}_4^{\,2} = 4\vec{R}_{CM}^{\,2} \Longrightarrow R_{N_C 0}(4b; R_C). \tag{69.8}$$

2.7 Radial integration

In the eq. (67) there are three identical radial integrals of the form

$$I = \int_0^\infty R_{n0}(r)\; e^{(-\frac{\beta}{2}r^2)}\; r^2\; dr \tag{70}$$

where

$$R_{n0}(r) = \sqrt{\frac{2n!\; b^{3/2}}{(n+1/2)!}}\; L_n^{\frac{1}{2}}(br^2)\; e^{(-\frac{1}{2}br^2)} \tag{71}$$

is harmonic oscillator radial wave function, and L is the Laguerre polynomial. By inserting (71) to (70) one gets

$$I = \sqrt{\frac{2n!\; b^{3/2}}{(n+1/2)!}} \int_0^\infty L_n^{\frac{1}{2}}(br^2)\; e^{(-\frac{1}{2}\beta r^2 - \frac{1}{2}br^2)}\; r^2\; dr. \tag{72}$$

If we use the following formula (ref. 58)

$$\int_0^\infty e^{-st}\; t^\kappa\; L_n^\alpha(t)\; dt = \frac{\Gamma(\kappa+1)\;\Gamma(\alpha+n+1)}{n!\;\Gamma(\alpha+1)}\; s^{-\kappa-1}\; F(-n, \kappa+1; \alpha+1; \frac{1}{s}) \tag{73}$$

where

$$\kappa = 1/2,\; \alpha = 1/2,\; s = \frac{\beta+b}{2b},\; F(-n, \kappa; \kappa; -z) = (1+z)^n \tag{73.1}$$

the integral I can be written as

$$I = \sqrt{\frac{4b^{\frac{3}{2}}}{(\beta+b)^3}}\; \left(\frac{\beta-b}{\beta+b}\right)^n\; \sqrt{\frac{(n+\frac{1}{2})!}{n!}}. \tag{74}$$

Finally, using the results of chapter 2.6, the overlap integral becomes:

$$
\begin{aligned}
F^{S_C=0}_{NNPP}(R_C) &= \left(\frac{2\beta^{3/2}}{(1/2)!}\right)^{3/2} \sum_{\alpha_\nu,\alpha_\pi} X_{NNPP}(\alpha_\nu,\alpha_\pi) \\
&\times \sum_{p\le q} \frac{\sqrt{2}X(pq;\alpha_\nu)}{\sqrt{(1+\delta_{j_pj_q}\delta_{l_pl_q}\delta_{n_pn_q})}} \sum_{r\le s} \frac{\sqrt{2}X(rs;\alpha_\pi)}{\sqrt{(1+\delta_{j_rj_s}\delta_{l_rl_s}\delta_{n_rn_s})}} \\
&\times \sum_{\lambda_\nu\lambda_\pi} \langle \lambda_\nu S_\nu = 0 | j_p j_q\rangle \langle \lambda_\pi S_\pi = 0 | j_r j_s\rangle \\
&\times \sum_{n_\nu N_\nu} \langle n_\nu 0 N_\nu \lambda_\nu; \lambda_\nu | n_p l_p n_q l_q; \lambda_\nu\rangle \sum_{n_\pi N_\pi} \langle n_\pi 0 N_\pi \lambda_\pi; \lambda_\pi | n_r l_r n_s l_s; \lambda_\pi\rangle \\
&\times \sum_{n_C N_C} \langle n_C 0 N_C 0; 0 | N_\nu \lambda_\nu N_\pi \lambda_\pi; 0\rangle \; (\chi^{S_\nu=0}\chi^{S_\pi=0})_{S_C=0} \; R_{N_C0}(4b;R_C) \\
&\times \left(\sqrt{\frac{4b^{\frac{3}{2}}}{(\beta+b)^3}}\right)^3 \left(\frac{\beta-b}{\beta+b}\right)^{n_\nu+n_\pi+n_C} \sqrt{\frac{(n_\nu+\frac{1}{2})!(n_\pi+\frac{1}{2})!(n_C+\frac{1}{2})!}{n_\nu!\,n_\pi!\,n_C!}}.
\end{aligned}
\tag{75}
$$

This is the general expression for the overlap integral for spin-singlet contribution.

2.8 Formation amplitude

Up to now we have not explicitly written the LS-jj coupling coefficients. They appear in eq. (75) for the overlap integral and they are calculated in APPENDIX A.
With LS-jj coupling coefficients taken into account the reduced width amplitude (75) for the (NN)-(PP) contribution if $J_\nu \neq 0$ and $J_\pi \neq 0$ finally reads

$$
\begin{aligned}
F^{S_C=0}_{NNPP}(R_C) &= \left(\frac{2\beta^{3/2}}{(1/2)!}\right)^{3/2} \sum_{\alpha_\nu,\alpha_\pi} X_{NNPP}(\alpha_\nu,\alpha_\pi) \\
&\times \sum_{p\le q} \frac{\sqrt{2}X(pq;\alpha_\nu)}{\sqrt{(1+\delta_{j_pj_q}\delta_{l_pl_q}\delta_{n_pn_q})}} \sum_{r\le s} \frac{\sqrt{2}X(rs;\alpha_\pi)}{\sqrt{(1+\delta_{j_rj_s}\delta_{l_rl_s}\delta_{n_rn_s})}} \\
&\times \sum_{J_\nu=J_\pi} (-1)^{l_p+j_q+J_\nu+1/2} \left[\frac{(2j_p+1)(2j_q+1)}{2}\right]^{1/2} \begin{Bmatrix} J_\nu & j_p & j_q \\ \frac{1}{2} & l_q & l_p \end{Bmatrix} \\
&\times (-1)^{l_r+j_s+J_\pi+1/2} \left[\frac{(2j_r+1)(2j_s+1)}{2}\right]^{1/2} \begin{Bmatrix} J_\pi & j_r & j_s \\ \frac{1}{2} & l_s & l_r \end{Bmatrix} \qquad (86) \\
&\times \sum_{n_\nu N_\nu} \langle n_\nu 0 N_\nu L_\nu; J_\nu | n_p l_p n_q l_q; J_\nu\rangle \sum_{n_\pi N_\pi} \langle n_\pi 0 N_\pi L_\pi; J_\pi | n_r l_r n_s l_s; J_\pi\rangle \\
&\times \sum_{n_C N_C} \langle n_C 0 N_C 0; 0 | N_\nu L_\nu N_\pi L_\pi; 0\rangle \, (\chi^{S_\nu=0}\chi^{S_\pi=0})_{S_C=0} \, R_{N_C0}(4b;R_C) \\
&\times \left(\frac{4b^{3/2}}{(\beta+b)^3}\right)^{3/2} \left(\frac{\beta-b}{\beta+b}\right)^{n_\nu+n_\pi+n_C} \sqrt{\frac{(n_\nu+\frac{1}{2})!(n_\pi+\frac{1}{2})!(n_C+\frac{1}{2})!}{n_\nu!\, n_\pi!\, n_C!}}.
\end{aligned}
$$

For the case when $J_\nu = J_\pi = 0$ the (NN)-(PP) reduced width amplitude is

$$
\begin{aligned}
F^{S_C=0}_{NNPP}(R_C) = {} & \frac{2^3\beta^{9/4}}{(\pi)^{3/4}}\, X_{NNPP} \\
& \times \sum_{p\le q} \frac{\sqrt{2}X(pq;\alpha_\nu)}{\sqrt{(1+\delta_{j_pj_q}\delta_{l_pl_q}\delta_{n_pn_q})}} \sqrt{\frac{(2j_p+1)}{2(2l_p+1)}} \\
& \times \sum_{r\le s} \frac{\sqrt{2}X(rs;\alpha_\pi)}{\sqrt{(1+\delta_{j_rj_s}\delta_{l_rl_s}\delta_{n_rn_s})}} \sqrt{\frac{(2j_r+1)}{2(2l_r+1)}} \\
& \times \sum_{n_\nu N_\nu} \langle n_\nu 0 N_\nu 0; 0 | n_p l_p n_p l_p; 0\rangle \sum_{n_\pi N_\pi} \langle n_\pi 0 N_\pi 0; 0 | n_r l_r n_r l_r; 0\rangle \\
& \times \sum_{n_C N_C} \langle n_C 0 N_C 0; 0 | N_\nu 0 N_\pi 0; 0\rangle \, (\chi^{S_\nu=0}\chi^{S_\pi=0})_{S_C=0}\, R_{N_C0}(4b;R_C) \\
& \times \left(\frac{4b^{3/2}}{(\beta+b)^3}\right)^{3/2} \left(\frac{\beta-b}{\beta+b}\right)^{n_\nu+n_\pi+n_C} \sqrt{\frac{(n_\nu+\frac{1}{2})!(n_\pi+\frac{1}{2})!(n_C+\frac{1}{2})!}{n_\nu!\, n_\pi!\, n_C!}}.
\end{aligned}
\tag{87}
$$

In this case the weight factor X_{NNPP} is a constant and the sum over different two-particle states α_π, α_ν dissapears.

2.9 (NP)–(NP) contribution

So far we discussed the (NN)(PP) contribution to the four-particle cluster wave function and consequently to the alpha decay reduced width amplitude. In a complete analogy we can now describe (NP)(NP) contribution. However, this time we have two terms $(1,3)(2,4) - (1,4)(2,3)$ of the same form. The cluster wave function analogous to eq. (46) according to eq. (15) and (fig. 2.3) is

$$
\begin{aligned}
&\Psi^{NPNP}_{J_C=0}(\vec{r}_1\vec{r}_2\vec{r}_3\vec{r}_4) = \Psi^{NPNP}_{J_C=0}(13,24) - \Psi^{NPNP}_{J_C=0}(14,23) = \\
&\sum_{\alpha_\delta,\alpha_{\delta\prime}} X_{NPNP}(\alpha_\delta,\alpha_{\delta\prime}) \\
&\times \Bigg\{ \sum_{p\le r} \frac{X(pr;\alpha_\delta)}{\sqrt{2(1+\delta_{j_pj_r}\delta_{l_pl_r}\delta_{n_pn_r})}} \sum_{\lambda_\delta S_\delta} \langle \lambda_\delta S_\delta | j_p j_r\rangle \\
&\times \sum_{n_\delta l_\delta N_\delta L_\delta} \langle n_\delta l_\delta N_\delta L_\delta;\lambda_\delta | n_p l_p n_r l_r;\lambda_\delta\rangle \\
&\times [1-(-1)^{S_\delta+T_\delta+l_\delta}]\,[[\varphi_{n_\delta l_\delta}(\vec{r}_\delta)\times\varphi_{N_\delta L_\delta}(\vec{R}_\delta)]_{\lambda_\delta}\times\chi(1,3)]_{J_\delta M_\delta}\Bigg\} \\
&\times \Bigg\{ \sum_{q\le s} \frac{X(qs;\alpha_{\delta\prime})}{\sqrt{2(1+\delta_{j_qj_s}\delta_{l_ql_s}\delta_{n_qn_s})}} \sum_{\lambda_{\delta\prime}S_{\delta\prime}} \langle \lambda_{\delta\prime} S_{\delta\prime} | j_q j_s\rangle \\
&\times \sum_{n_{\delta\prime}l_{\delta\prime}N_{\delta\prime}L_{\delta\prime}} \langle n_{\delta\prime}l_{\delta\prime}N_{\delta\prime}L_{\delta\prime};\lambda_{\delta\prime} | n_q l_q n_s l_s;\lambda_{\delta\prime}\rangle \\
&\times [1-(-1)^{S_{\delta\prime}+T_{\delta\prime}+l_{\delta\prime}}]\,[[\varphi_{n_{\delta\prime}l_{\delta\prime}}(\vec{r}_{\delta\prime})\times\varphi_{N_{\delta\prime}L_{\delta\prime}}(\vec{R}_{\delta\prime})]_{\lambda_{\delta\prime}}\times\chi(2,4)]_{J_{\delta\prime}M_{\delta\prime}}\Bigg\}_{J_C=0} \\
&- \sum_{\alpha'_\delta,\alpha'_{\delta\prime}} X_{NPNP}(\alpha'_\delta,\alpha'_{\delta\prime}) \\
&\times \Bigg\{ \sum_{p\le s} \frac{X(ps;\alpha'_\delta)}{\sqrt{2(1+\delta_{j_pj_s}\delta_{l_pl_s}\delta_{n_pn_s})}} \sum_{\lambda'_\delta S'_\delta} \langle \lambda'_\delta S'_\delta | j_p j_s\rangle \\
&\times \sum_{n'_\delta l'_\delta N'_\delta L'_\delta} \langle n_\delta l_\delta N_\delta L_\delta;\lambda'_\delta | n_p l_p n_s l_s;\lambda'_\delta\rangle \\
&\times [1-(-1)^{S'_\delta+T'_\delta+l'_\delta}]\,[[\varphi_{n'_\delta l'_\delta}(\vec{r}'_\delta)\times\varphi_{N'_\delta L'_\delta}(\vec{R}'_\delta)]_{\lambda'_\delta}\times\chi(1,4)]_{J'_\delta M'_\delta}\Bigg\} \\
&\times \Bigg\{ \sum_{q\le r} \frac{X(qr;\alpha'_{\delta\prime})}{\sqrt{2(1+\delta_{j_qj_r}\delta_{l_ql_r}\delta_{n_qn_r})}} \sum_{\lambda'_{\delta\prime}S'_{\delta\prime}} \langle \lambda'_{\delta\prime} S'_{\delta\prime} | j_q j_r\rangle \\
&\times \sum_{n'_{\delta\prime}l'_{\delta\prime}N'_{\delta\prime}L'_{\delta\prime}} \langle n'_{\delta\prime}l'_{\delta\prime}N'_{\delta\prime}L'_{\delta\prime};\lambda'_{\delta\prime} | n_q l_q n_r l_r;\lambda'_{\delta\prime}\rangle \\
&\times [1-(-1)^{S'_{\delta\prime}+T'_{\delta\prime}+l'_{\delta\prime}}]\,[[\varphi_{n'_{\delta\prime}l'_{\delta\prime}}(\vec{r}'_{\delta\prime})\times\varphi_{N'_{\delta\prime}L'_{\delta\prime}}(\vec{R}'_{\delta\prime})]_{\lambda'_{\delta\prime}}\times\chi(2,3)]_{J'_{\delta\prime}M'_{\delta\prime}}\Bigg\}_{J'_C=0}.
\end{aligned}
\tag{88}
$$

In formula (88) the expression of the type

$$\Psi_{J_C} \sim \Big\{ [[\varphi_{n_\delta l_\delta}(\vec{r}_\delta) \times \varphi_{N_\delta L_\delta}(\vec{R}_\delta)]_{\lambda_\delta} \times \chi^{S_\delta}]_{J_\delta=1} \times [[\varphi_{n_{\delta'} l_{\delta'}}(\vec{r}_{\delta'}) \times \varphi_{N_{\delta'} L_{\delta'}}(\vec{R}_{\delta'})]_{\lambda_{\delta'}} \times \chi^{S_{\delta'}}]_{J_{\delta'}=1} \Big\}_{J_C=0} \tag{89}$$

appear. It includes implicitly the following couplings:

$$\begin{aligned} A &= [\big((Y_{l_\delta} Y_{L_\delta})_{\lambda_\delta} \chi^{S_\delta}\big)_{J_\delta} \times \big((Y_{l_{\delta'}} Y_{L_{\delta'}})_{\lambda_{\delta'}} \chi^{S_{\delta'}})_{J_{\delta'}}\big)]_{J_C} \\ &= \sum_{J_C M_{J_C}} \langle (\lambda_\delta S_\delta) J_\delta \ (\lambda_{\delta'} S_{\delta'}) J_{\delta'}; J_C | (\lambda_\delta \lambda_{\delta'}) L_C \ (S_\delta S_{\delta'}) S_C; J_C \rangle \\ &\times \Big\{ [(Y_{l_\delta} Y_{L_\delta})_{\lambda_\delta} (Y_{l_{\delta'}} Y_{L_{\delta'}})_{\lambda_{\delta'}}]_{L_C} \ (\chi^{S_\delta} \chi^{S_{\delta'}})_{S_C} \Big\}_{J_C}. \end{aligned} \tag{90}$$

After spherical harmonics recoupling

$$\begin{aligned} &[(Y_{l_\delta} Y_{L_\delta})_{\lambda_\delta} (Y_{l_{\delta'}} Y_{L_{\delta'}})_{\lambda_{\delta'}}]_{L_C} = \\ &\sum_{L_C M_{L_C}} \langle (l_\delta L_\delta)\lambda_\delta \ (l_{\delta'} L_{\delta'})\lambda_{\delta'}; L_C | (l_\delta l_{\delta'}) l_C (L_\delta L_{\delta'}) L_C; L_C \rangle \\ &\times [(Y_{l_\delta} Y_{l_{\delta'}})_{l_C} \ (Y_{L_\delta} \ Y_{L_{\delta'}})_{L_C}]_{L_C} \end{aligned} \tag{91}$$

and using the condition

$$l_C = 0 \ \ J_C = 0 \ \ S_C = 0 \Longrightarrow L_C = 0 \tag{92}$$

the cluster wave function results in two terms of the type (see eq. (88)) where

$$\Psi^{NPNP}_{J_C=0}(13,24) = \sum_{\alpha_\delta,\alpha_{\delta'}} X_{NPNP}(\alpha_\delta,\alpha_{\delta'})$$

$$\times \Big\{ \sum_{p\le s} \frac{X(ps;\alpha_\delta)}{\sqrt{2(1+\delta_{j_pj_s}\delta_{l_pl_s}\delta_{n_pn_s})}} \sum_{\lambda_\delta S_\delta} \langle \lambda_\delta S_\delta | j_p j_s\rangle$$

$$\times \sum_{n_\delta l_\delta N_\delta L_\delta} \langle n_\delta l_\delta N_\delta L_\delta; \lambda_\delta | n_p l_p n_s l_s; \lambda_\delta\rangle [1-(-1)^{S_\delta+T_\delta+l_\delta}]$$

$$\times \sum_{q\le r} \frac{X(qr;\alpha_{\delta'})}{\sqrt{2(1+\delta_{j_qj_r}\delta_{l_ql_r}\delta_{n_qn_r})}} \sum_{\lambda_{\delta'} S_{\delta'}} \langle \lambda_{\delta'} S_{\delta'} | j_q j_r\rangle \qquad (93)$$

$$\times \sum_{n_{\delta'} l_{\delta'} N_{\delta'} L_{\delta'}} \langle n_{\delta'} l_{\delta'} N_{\delta'} L_{\delta'}; \lambda_{\delta'} | n_q l_q n_r l_r; \lambda_{\delta'}\rangle [1-(-1)^{S_{\delta'}+T_{\delta'}+l_{\delta'}}]$$

$$\times \langle (\lambda_\delta S_\delta) J_\delta \ (\lambda_{\delta'} S_{\delta'}) J_{\delta'}; J_C = 0 | (\lambda_\delta \lambda_{\delta'}) 0 \ (S_\delta S_{\delta'}) S_C; J_C = 0\rangle$$

$$\times \langle ((l_\delta L_\delta)\lambda_\delta \ (l_{\delta'} L_{\delta'})\lambda_{\delta'}; 0 | (l_\delta l_{\delta'}) 0 \ (L_\delta L_{\delta'}) 0; 0\rangle \ [\chi^{S_\delta}\chi^{S_{\delta'}}]_0$$

$$\times [(Y_{l_\delta} Y_{l_{\delta'}})_{l_c=0} \ (Y_{L_\delta} Y_{L_{\delta'}})_{L_C=0}]_{L_C=0}$$

$$\times R_{n_\delta l_\delta}(r_\delta) \ R_{N_\delta L_\delta}(R_\delta) \ R_{n_{\delta'} l_{\delta'}}(r_{\delta'}) \ R_{N_{\delta'} L_{\delta'}}(R_{\delta'}) \Big\}_{J_C=0}.$$

The next step is to make Moshinsky-Talmi transformation , from the $N_\nu L_\nu$, $N_\pi L_\pi$ pair centre-of-mass coordinates to the $n_C l_C; N_C L_C$ cluster relative and centre-of-mass coordinates

$$R_{N_\delta L_\delta} \ R_{N_{\delta'} L_{\delta'}} \ (Y_{L_\delta} \ Y_{L_{\delta'}})_{L_C=0} = \sum_{n_C l_C N_C L_C} \langle n_C l_C N_C L_C; J_C | N_\delta L_\delta N_{\delta'} L_{\delta'}; J_C\rangle$$

$$\times R_{n_C l_C}(r_C) \ R_{N_C L_C}(R_C) \ (Y_{l_C} \ Y_{L_C})_{L_C} \qquad (94.1)$$

that is equivalent of the eq. (51) . Or

$$R_{N_\delta L_\delta}\ R_{N_{\delta'} L_{\delta'}}\ (Y_{L_\delta} Y_{L_{\delta'}})_0 =$$

$$\sum_{n_C N_C} \langle n_C 0 N_C 0; 0 | N_\delta 0 N_{\delta'} 0; 0\rangle\ R_{n_C 0}(r_C)\ R_{N_C 0}(R_C)\ (Y_0\ Y_0)_0. \tag{94.2}$$

The cluster wave function after Moshinsky-Talmi transformation is built of two terms : $(1,3)(2,4) - (1,4)(2,3)$ of the same form where:

$$\begin{aligned}
&\Psi^{NPNP}_{J_C=0}(13,24) = \sum_{\alpha_\delta, \alpha_{\delta'}} X_{NPNP}(\alpha_\delta, \alpha_{\delta'}) \\
&\times \Big\{ \sum_{p \le s} \frac{X(ps;\alpha_\delta)}{\sqrt{2(1+\delta_{j_p j_s}\delta_{l_p l_s}\delta_{n_p n_s})}} \sum_{\lambda_\delta S_\delta} \langle \lambda_\delta S_\delta | j_p j_s \rangle \\
&\times \sum_{n_\delta l_\delta N_\delta L_\delta} \langle n_\delta l_\delta N_\delta L_\delta; \lambda_\delta | n_p l_p n_s l_s; \lambda_\delta \rangle [1 - (-1)^{S_\delta + T_\delta + l_\delta}] \\
&\times \sum_{q \le r} \frac{X(qr;\alpha_{\delta'})}{\sqrt{2(1+\delta_{j_q j_r}\delta_{l_q l_r}\delta_{n_q n_r})}} \sum_{\lambda_{\delta'} S_{\delta'}} \langle \lambda_{\delta'} S_{\delta'} | j_q j_r \rangle \\
&\times \sum_{n_{\delta'} l_{\delta'} N_{\delta'} L_{\delta'}} \langle n_{\delta'} l_{\delta'} N_{\delta'} L_{\delta'}; \lambda_{\delta'} | n_q l_q n_r l_r; \lambda_{\delta'} \rangle [1 - (-1)^{S_{\delta'} + T_{\delta'} + l_{\delta'}}] \\
&\times \langle (\lambda_\delta S_\delta) J_\delta\ (\lambda_{\delta'} S_{\delta'}) J_{\delta'}; 0 | (\lambda_\delta \lambda_{\delta'}) 0\ (S_\delta S_{\delta'}) 0; 0 \rangle \\
&\times \langle (l_\delta L_\delta) \lambda_\delta\ (l_{\delta'} L_{\delta'}) \lambda_{\delta'}; 0 | (l_\delta l_{\delta'}) 0\ (L_\delta L_{\delta'}) 0; 0 \rangle \\
&\times \sum_{n_C N_C} \langle n_C 0 N_C 0; 0 | N_\delta L_\delta N_{\delta'} L_{\delta'}; 0 \rangle\ [\chi^{S_\delta}\ \chi^{S_{\delta'}}]_0 \\
&\times R_{n_C 0}(r_C)\ R_{N_C 0}(R_C)\ R_{n_\delta l_\delta}(r_\delta)\ R_{n_{\delta'} l_{\delta'}}(r_{\delta'})\ Y_0\ Y_0\ (Y_{l_\delta}\ Y_{l_{\delta'}})_0 \Big\}_{J_C=0}.
\end{aligned} \tag{95}$$

Both terms in $\Psi^{NPNP}_{J_C=0}$ will result in the same kind of reduced width amplitude and after summing of the two, one gets

$$
\begin{aligned}
F^{S_C=0}_{NPNP}(R_C) = & \left(\frac{2\beta^{3/2}}{(1/2)!}\right)^{3/2} \sum_{\alpha_\delta,\alpha'_\delta} X_{NPNP}(\alpha_\delta,\alpha'_\delta) \\
& \times \sum_{p\le r} \frac{\sqrt{2}X(pr;\alpha_\delta)}{\sqrt{(1+\delta_{j_pj_r}\delta_{l_pl_r}\delta_{n_pn_r})}} \sum_{q\le s} \frac{\sqrt{2}X(qs;\alpha'_\delta)}{\sqrt{(1+\delta_{j_qj_s}\delta_{l_ql_s}\delta_{n_qn_s})}} \\
& \times \sum_{\lambda_\delta\lambda'_\delta} \langle \lambda_\delta S_\delta | j_p j_r\rangle \langle \lambda'_\delta S'_\delta | j_q j_s\rangle \\
& \times \sum_{n_\delta N_\delta} \langle n_\delta 0 N_\delta \lambda_\delta; \lambda_\delta | n_p l_p n_r l_r; \lambda_\delta\rangle \sum_{n'_\delta N'_\delta} \langle n'_\delta 0 N'_\delta \lambda'_\delta; \lambda'_\delta | n_q l_q n_s l_s; \lambda'_\delta\rangle \\
& \times \sum_{n_C N_C} \langle n_C 0 N_C 0; 0 | N_\delta \lambda_\delta N'_\delta \lambda'_\delta; 0\rangle \\
& \times (\chi^{S_\delta}(1,3)\, \chi^{S_{\delta'}}(2,4))_{S_C=0}\ R_{N_C0}(4b;R_C) \\
& \times \left(\sqrt{\frac{4b^{\frac{3}{2}}}{(\beta+b)^3}}\right)^3 \left(\frac{\beta-b}{\beta+b}\right)^{n_\delta+n_{\delta'}+n_C} \sqrt{\frac{(n_\delta+\frac{1}{2})!(n_{\delta'}+\frac{1}{2})!(n_C+\frac{1}{2})!}{n_\delta!\, n_{\delta'}!\, n_C!}} \\
& - \left(\frac{2\beta^{3/2}}{(1/2)!}\right)^{3/2} \sum_{\alpha'_\delta,\alpha'_{\delta'}} X_{NPNP}(\alpha'_\delta,\alpha'_{\delta'}) \\
& \times \sum_{p\le s} \frac{\sqrt{2}X(ps;\alpha'_\delta)}{\sqrt{(1+\delta_{j_pj_s}\delta_{l_pl_s}\delta_{n_pn_s})}} \sum_{q\le r} \frac{\sqrt{2}X(qr;\alpha'_{\delta'})}{\sqrt{(1+\delta_{j_qj_r}\delta_{l_ql_r}\delta_{n_qn_r})}} \\
& \times \sum_{\lambda'_\delta\lambda'_{\delta'}} \langle \lambda'_\delta S'_\delta | j_p j_s\rangle \langle \lambda'_{\delta'} S'_{\delta'} | j_q j_r\rangle \\
& \times \sum_{n'_\delta N'_\delta} \langle n'_\delta 0 N'_\delta \lambda'_\delta; \lambda'_\delta | n_p l_p n_s l_s; \lambda'_\delta\rangle \sum_{n'_{\delta'} N'_{\delta'}} \langle n'_{\delta'} 0 N'_{\delta'} \lambda'_{\delta'}; \lambda'_{\delta'} | n_q l_q n_r l_r; \lambda'_{\delta'}\rangle \\
& \times \sum_{n'_C N'_C} \langle n'_C 0 N'_C 0; 0 | N'_\delta \lambda'_\delta N'_{\delta'} \lambda'_{\delta'}; 0\rangle \\
& \times (\chi^{S_\delta}(1,4)\, \chi^{S_{\delta'}}(2,3))_{S_C=0}\ R_{N_C0}(4b;R_C) \\
& \times \left(\sqrt{\frac{4b^{\frac{3}{2}}}{(\beta+b)^3}}\right)^3 \left(\frac{\beta-b}{\beta+b}\right)^{n'_\delta+n'_{\delta'}+n'_C} \sqrt{\frac{(n'_\delta+\frac{1}{2})!(n'_{\delta'}+\frac{1}{2})!(n'_C+\frac{1}{2})!}{n'_\delta!\, n'_{\delta'}!\, n'_C!}}.
\end{aligned}
\tag{96}
$$

It is evident that one can write down both terms in more compact way (as the summations over different indexes have the same limits).

So the reduced width amplitude may be written as:

$$
\begin{aligned}
F^{S_C=0}_{NPNP}(R_C) = & \left(\frac{2\beta^{3/2}}{(1/2)!}\right)^{3/2} \sum_{\alpha_\delta,\alpha'_\delta} X_{NPNP}(\alpha_\delta,\alpha'_\delta) \\
& \times \sum_{p\le r} \frac{\sqrt{2}X(pr;\alpha_\delta)}{\sqrt{(1+\delta_{j_pj_r}\delta_{l_pl_r}\delta_{n_pn_r})}} \sum_{q\le s} \frac{\sqrt{2}X(qs;\alpha'_\delta)}{\sqrt{(1+\delta_{j_qj_s}\delta_{l_ql_s}\delta_{n_qn_s})}} \\
& \times \sum_{\lambda_\delta\lambda'_\delta} \langle \lambda_\delta S_\delta | j_p j_r\rangle \langle \lambda'_\delta S'_\delta | j_q j_s\rangle \\
& \times \sum_{n_\delta N_\delta} \langle n_\delta 0 N_\delta \lambda_\delta; \lambda_\delta | n_p l_p n_r l_r; \lambda_\delta\rangle \sum_{n'_\delta N'_\delta} \langle n'_\delta 0 N'_\delta \lambda'_\delta; \lambda'_\delta | n_q l_q n_s l_s; \lambda'_\delta\rangle \\
& \times \sum_{n_C N_C} \langle n_C 0 N_C 0; 0 | N_\delta \lambda_\delta N'_\delta \lambda'_\delta; 0\rangle \; R_{N_C 0}(4b;R_C) \\
& \times \{[\chi^{S_\delta}(1,3)\,\chi^{S_{\delta'}}(2,4)]_{S_C=0} - [\chi^{S_\delta}(1,4)\,\chi^{S_{\delta'}}(2,3)]_{S_C=0}\} \\
& \times \left(\sqrt{\frac{4b^{\frac{3}{2}}}{(\beta+b)^3}}\right)^3 \left(\frac{\beta-b}{\beta+b}\right)^{n_\delta+n_{\delta'}+n_C} \sqrt{\frac{(n_\delta+\frac{1}{2})!(n_{\delta'}+\frac{1}{2})!(n_C+\frac{1}{2})!}{n_\delta!\,n_{\delta'}!\,n_C!}}.
\end{aligned}
\tag{97}
$$

From the angular integration, which is identical to the (NN)-(PP) case, comes the condition

$$
\int d\hat{r}_\delta \; Y_{l_\delta m_\delta}(\hat{r}_\delta) = \sqrt{4\pi}\; \delta(l_\delta,0)\; \delta(m_\delta,0).
$$

Taking into account the phase factor $[1-(-1)^{S_\delta+T_\delta+l_\delta}]$ from eq. (95),we are left with two possible choices for (NP) pairs:

1) Spin-triplet for $T=0$

$$S_\delta=1,\ l_\delta=0,\ \lambda_\delta=0,\ J_\delta=1$$

2) Spin-singlet for $T=1$

$$S_\delta=0,\ l_\delta=0,\ \lambda_\delta=0,\ J_\delta=0. \tag{98}$$

The later (spin-singlet) contribution is completely analogous to the (spin-singlet) contribution of (NN)-(PP) cluster.
The spin part of eq. (97),(see fig. 2.3) is therefore

$$\begin{aligned}
&\{[\chi^{S_\delta}(1,3)\ \chi^{S_{\delta'}}(2,4)]_0-[\chi^{S_\delta}(1,4)\ \chi^{S_{\delta'}}(2,3)]_0\}=\\
&\{[\chi_{11}(1,3)\chi_{1-1}(2,4)]_{00}+[\chi_{1-1}(1,3)\chi_{11}(2,4)]_{00}\}\\
&-\{[\chi_{10}(1,4)\chi_{10}(2,3)]_{00}-[\chi_{00}(1,4)\chi_{00}(2,3)]_{00}\}\\
&=\langle 1\ 1\ 1\ -1|0\ 0\rangle\chi_{11}(1,3)\chi_{1-1}(2,4)+\langle 1\ -1\ 1\ 1|0\ 0\rangle\chi_{1-1}(1,3)\chi_{11}(2,4)\\
&-\langle 1\ 0\ 1\ 0|0\ 0\rangle\chi_{10}(1,4)\chi_{10}(2,3)+\langle 0\ 0\ 0\ 0|0\ 0\rangle\chi_{00}(1,4)\chi_{00}(2,3)\\
&=\frac{1}{\sqrt{3}}\{\chi_{11}(1,3)\chi_{1-1}(2,4)+\chi_{1-1}(1,3)\chi_{11}(2,4)+\chi_{10}(1,4)\chi_{10}(2,3)\}\\
&+\chi_{00}(1,4)\chi_{00}(2,3)=\frac{1}{\sqrt{3}}\chi^1(N,P)\chi^1(N,P)+\chi^0(N,P)\chi^0(N,P).
\end{aligned} \tag{99}$$

The notation is the following (see ch.2.3):

$$\chi^1(N,P)=\begin{cases}\chi_{11}(N,P)\\ \chi_{10}(N,P)\\ \chi_{1-1}(N,P)\end{cases}$$

and

$$\chi^0(N,P)=\chi_{00}(N,P).$$

2.10 Spin-triplet part of (N-P)–(N-P)

From eq. (99) it is evident that there are two different contributions, with respect to spin, to (NN)-(PP) cluster; spin-singlet and spin-triplet. Now we can apply the eq. (95) for the (NP)-(NP) cluster to the case $S_\delta = S_{\delta'} = 1$, which we call for spin-triplet contribution (where "triplet" denotes two-particle spins). The wave function of the triplet contribution therefore may be written as:

$$\Psi^{triplet}_{J_C=0}(\vec{r}_1\vec{r}_2\vec{r}_3\vec{r}_4) = X_{NPNP}$$

$$\times \Big\{ \sum_{p\le s} \frac{\sqrt{2}X(ps;\alpha_\delta)}{\sqrt{(1+\delta_{j_pj_s}\delta_{l_pl_s}\delta_{n_pn_s})}} \langle \lambda_\delta = 0\ S_\delta = 1 | j_p j_s\rangle \sum_{n_\delta N_\delta} \langle n_\delta 0 N_\delta 0; 0 | n_p l_p n_s l_s; 0\rangle$$

$$\times \sum_{q\le r} \frac{\sqrt{2}X(qr;\alpha_{\delta'})}{\sqrt{(1+\delta_{j_qj_r}\delta_{l_ql_r}\delta_{n_qn_r})}} \langle \lambda_{\delta'} = 0\ S_{\delta'} = 1 | j_q j_r\rangle \sum_{n_{\delta'} N_{\delta'}} \langle n_{\delta'} 0 N_{\delta'} 0; 0 | n_q l_q n_r l_r; 0\rangle$$

$$\times \langle (01)1\ (01)1; 0 | (00)0\ (11)0; 0\rangle\ \langle (00)0\ (00)0; 0 | (00)0\ (00)0; 0\rangle$$

$$\times \sum_{n_C N_C} \langle n_C\ 0\ N_C\ 0; 0 | N_\delta\ 0\ N_{\delta'}\ 0; 0\rangle \times [\chi^{S_\delta=1}\ \chi^{S_{\delta'}=1}]_0$$

$$\times\ R_{n_C0}(r_C)\ R_{N_C0}(R_C)\ R_{n_\delta 0}(r_\delta)\ R_{n_{\delta'}0}(r_{\delta'})\ Y_0\ Y_0\ Y_0\ Y_0 \Big\}_{J_C=0}. \tag{100}$$

With the help of the relations

$$\begin{aligned} \langle (01)1(01)1; 0 | (00)0(11)0; 0\rangle &= 1 \\ \langle (00)0(00)0; 0 | (00)0(00)0; 0\rangle &= 1 \end{aligned} \tag{101}$$

it results in

$$\Psi^{triplet}_{J_C=0}(\vec{r}_1\vec{r}_2\vec{r}_3\vec{r}_4) = X_{NPNP}$$

$$\times \Big\{ \sum_{p\le s} \frac{\sqrt{2}X(ps;\alpha_\delta)}{\sqrt{(1+\delta_{j_pj_s}\delta_{l_pl_s}\delta_{n_pn_s})}} \langle \lambda_\delta = 0\ S_\delta = 1 | j_p\ j_s\rangle \sum_{n_\delta N_\delta} \langle n_\delta 0 N_\delta 0; 0 | n_p l_p n_s l_s; 0\rangle$$

$$\times \sum_{q\le r} \frac{\sqrt{2}X(qr;\alpha_{\delta'})}{\sqrt{(1+\delta_{j_qj_r}\delta_{l_ql_r}\delta_{n_qn_r})}} \langle \lambda_{\delta'} = 0\ S_{\delta'} = 1 | j_q\ j_r\rangle \sum_{n_{\delta'} N_{\delta'}} \langle n_{\delta'} 0 N_{\delta'} 0; 0 | n_q l_q n_r l_r; 0\rangle$$

$$\times \sum_{n_C N_C} \langle n_C 0 N_C 0; 0 | N_\delta 0 N_{\delta'} 0; 0\rangle \times [\chi^{S_\delta=1}\ \chi^{S_{\delta'}=1}]_0$$

$$\times R_{n_C0}(r_C)\ R_{N_C0}(R_C)\ R_{n_\delta 0}(r_\delta)\ R_{n_{\delta'}0}(r_{\delta'})\ Y_0\ Y_0\ Y_0\ Y_0 \Big\}_{J_C=0}. \tag{102}$$

This expression exactly coincides with the formula for spin-singlet (NN)-(PP) contribution, except for LS-jj coupling coefficients, which are this time

$$\langle \lambda_\delta = 0\ S_\delta = 1 | j_p j_s\rangle \qquad \langle \lambda_{\delta'} = 0\ S_{\delta'} = 1 | j_q j_r\rangle$$

compared to

$$\langle \lambda_\nu = 0\ S_\nu = 0 | j_p j_q\rangle \qquad \langle \lambda_\pi = 0\ S_\pi = 0 | j_r j_s\rangle$$

or

$$\langle \lambda_\nu\ S_\nu = 0 | j_p j_q\rangle \qquad \langle \lambda_\pi\ S_\pi = 0 | j_r j_s\rangle.$$

By the analogy with (NN)-(PP) derivation we write the final expression for spin-triplet (NP)-(NP) contribution to the alpha-decay absolute reduced width as

$$F^{triplet}_{NPNP}(R_C) = (\frac{2^3\beta^{9/4}}{\pi^{3/4}})\ X_{NPNP}$$

$$\times \Big\{ \sum_{p\le s} \frac{\sqrt{2}X(ps;\alpha_\delta)}{\sqrt{(1+\delta_{j_pj_s}\delta_{l_pl_s}\delta_{n_pn_s})}}\ \langle \lambda_\delta = 0\ S_\delta = 1 | j_p j_s\rangle$$

$$\times \sum_{q\le r} \frac{\sqrt{2}X(qr;\alpha_{\delta'})}{\sqrt{(1+\delta_{j_qj_r}\delta_{l_ql_r}\delta_{n_qn_r})}}\ \langle \lambda_{\delta'} = 0\ S_{\delta'} = 1 | j_q j_r\rangle$$

$$\times \sum_{n_\delta N_\delta} \langle n_\delta 0 N_\delta 0; 0 | n_p l_p n_s l_s; 0\rangle \sum_{n_{\delta'} N_{\delta'}} \langle n_{\delta'} 0 N_{\delta'} 0; 0 | n_q l_q n_r l_r; 0\rangle$$

$$\times \sum_{n_C N_C} \langle n_C 0 N_C 0; 0 | N_\delta 0 N_{\delta'} 0; 0\rangle\ R_{N_C 0}(4b; R_C)\ [\chi^{S_\delta}\ \chi^{S_{\delta'}}]_0$$

$$\times \left(\sqrt{\frac{4b^{\frac{3}{2}}}{(\beta+b)^3}}\right)^{3/2} \left(\frac{\beta-b}{\beta+b}\right)^{n_\delta+n_{\delta'}+n_C} \sqrt{\frac{(n_\delta+\frac{1}{2})!(n_{\delta'}+\frac{1}{2})!(n_C+\frac{1}{2})!}{n_\delta!\ n_{\delta'}!\ n_C!}}\ \Big\} \tag{103}$$

where

$$\langle \lambda = 0\ S = 1 | j_1 j_2\rangle = (-1)^{l+j_1+3/2} \sqrt{\frac{(2j_1+1)(2j_2+1)}{(2l+1)}} \begin{Bmatrix} j_2 & j_1 & 1 \\ \frac{1}{2} & \frac{1}{2} & l \end{Bmatrix}. \tag{104}$$

We have now calculated all parts that contribute to the overlap integral from both (NN)(PP) (spin-singlet) and (NP)(NP) (spin-singlet and spin-triplet) terms. It is easy to see that they have essentially the same form. We can write

$$\mathbf{\Omega_{R_C}} = \left(\frac{2^3\beta^{9/4}}{\pi^{3/4}}\right) \Bigg\{ \sum_{e\le h} \frac{\sqrt{2}X(eh;\alpha_\epsilon)}{\sqrt{(1+\delta_{j_e j_h}\delta_{l_e l_h}\delta_{n_e n_h})}} \sum_{f\le g} \frac{\sqrt{2}X(fg;\alpha_{\epsilon'})}{\sqrt{(1+\delta_{j_f j_g}\delta_{l_f l_g}\delta_{n_f n_g})}}$$

$$\times \langle 0\ S_\epsilon | j_e\ j_h\rangle\ \langle 0\ S_{\epsilon'} | j_f\ j_g\rangle$$

$$\times \sum_{n_\epsilon N_\epsilon} \langle n_\epsilon 0 N_\epsilon 0; 0 | n_e l_e n_h l_h; 0\rangle \sum_{n_{\epsilon'} N_{\epsilon'}} \langle n_{\epsilon'} 0 N_{\epsilon'} 0; 0 | n_f l_f n_g l_g; 0\rangle$$

$$\times \sum_{n_C N_C} \langle n_C 0 N_C 0; 0 | N_\epsilon \lambda_\epsilon N_{\epsilon'} \lambda_{\epsilon'}; 0\rangle\ R_{N_C 0}(4b; R_C)\ [\chi^{S_\epsilon}\ \chi^{S_{\epsilon'}}]_0$$

$$\times \left(\sqrt{\frac{4b^{\frac{3}{2}}}{(\beta+b)^3}}\right)^{3/2} \left(\frac{\beta-b}{\beta+b}\right)^{n_\epsilon+n_{\epsilon'}+n_C} \sqrt{\frac{(n_\epsilon+\frac{1}{2})!(n_{\epsilon'}+\frac{1}{2})!(n_C+\frac{1}{2})!}{n_\epsilon!\ n_{\epsilon'}!\ n_C!}} \Bigg\} \tag{105}$$

where

$$(\epsilon,\epsilon') \equiv \begin{cases} (\nu,\pi) & \text{for NNPP} \\ (\delta,\delta') & \text{for NPNP} \end{cases}$$

which helps us to express the final formula in a very compact form.

2.11 Alpha–decay absolute reduced width and halflife

Finally one can collect together all the pieces that make reduced width. So in terms of overlap integrals Ω_{R_C} it will be

$$F(R_C) = X_{NNPP}\ \frac{6}{\sqrt{24}}\ \mathbf{\Omega_{R_C}}^{NNPP}(\lambda_\epsilon = 0\ S_\epsilon = 0\ ;\lambda_{\epsilon'} = 0\ S_{\epsilon'} = 0)$$

$$+X_{NPNP}\ \frac{6}{\sqrt{24}}\ [\ \mathbf{\Omega_{R_C}}^{NPNP}(\lambda_\epsilon = 0\ S_\epsilon = 0\ ;\lambda_{\epsilon'} = 0\ S_{\epsilon'} = 0) \qquad (106)$$

$$+\frac{1}{\sqrt{3}}\ \mathbf{\Omega_{R_C}}^{NPNP}(\lambda_\epsilon = 0\ S_\epsilon = 1\ ;\lambda_{\epsilon'} = 0\ S_{\epsilon'} = 1)].$$

The factor $\frac{6}{\sqrt{24}}$ commes from the antisymmetrization of the four-particle wave function (notice that fig. 2.4 consists of six contributions equivalent to fig. 2.3). The final expression is a fairly simple and compact one. The (singlet)-(singlet) contribution from both NN-PP and NP-NP follows the same formula as the eq. (87). The (triplet)-(triplet) contribution from NP-NP is practically the same, except for the 6-j and spin function which are different. Radial parts are the ones which correspond to NN, PP, and PN pairs, respectively.

From the overlap integral F(R) it is easy to get the alpha-decay absolute reduced width

$$\gamma_L(R) = (\frac{\hbar^2\ R}{2\mu})\ \ F(R)$$

and the α-decay half-life defined before by (eqs. 21.1 and 21.2)

$$T_{1/2} = \frac{\hbar\ ln2}{2\ P_L\ \gamma_L^2}\ .$$

3. RESULTS

The main strategy of our calculations of α-decay width as well as of those done before (ref. 33–36) was to divide the α-decay process into the following two steps:

1. Cluster formation
2. Tunnelling through the barrier

According to that basic assumption the derivation of the α-decay absolute reduced widths as well as the half-lives is made. We are using harmonic oscillator single–particle radial wave functions which enables us to make overlap integral analytically. Formulas (106), i.e. (87) and (103) are used to create Fortran programs for the calculation of the ^{212}Po α-decay.

It is important to remember the following two points. For quite a long time it was recognised that the configuration mixing plays a key role in the process of α-decay and α-clustering (ref. 62, 83, 113). That fact is clearly demonstrated in present calculation.

Configuration mixing enables protons and neutrons otherwise filling different shells to interact. We are showing that the proton-neutron interaction plays a very significant role in the process of α-cluster formation and thereafter in the α-decay process.
Calculation of the tunneling through the barrier is based on (ref. 12). To obtain the α-decay half-life we have to go through the several intermediate steps:

1. Calculate proton and neutron single-particle levels by program SPLEV

2. Calculate two-particle PP, NN and PN states by PPSDI, NNSDI and PNSDI programs

3. Calculate α-decay reduced width by program ALPHA.

4. Calculate Coulomb wave functions (penetrability) and α-decay half-life by program HALFLIFE

Listings of all programs are given in APPENDIX B

3.1 Single–particle wave functions

In all our earlier calculations (ref. 33-36) single-particle wave functions have the Woods–Saxon radial part. As a consequence, the 11-dimensional overlap integral was calculated numerically, what was extremely time-consuming, 10–20 hours of CPU-time at VAX11/780 computer. In our present work the integration was performed analytically using the harmonic–oscillator radial wave functions and Moshinsky–Talmi transformation.
Single-partcle energies are taken from (ref. 22) or calculated by program SPLEV (for high-lying configurations). To test our harmonic-oscillator radial wave functions we compare them with Woods–Saxon radial wave functions. Here are the plots of both radial wave functions for the corresponding quantum numbers.

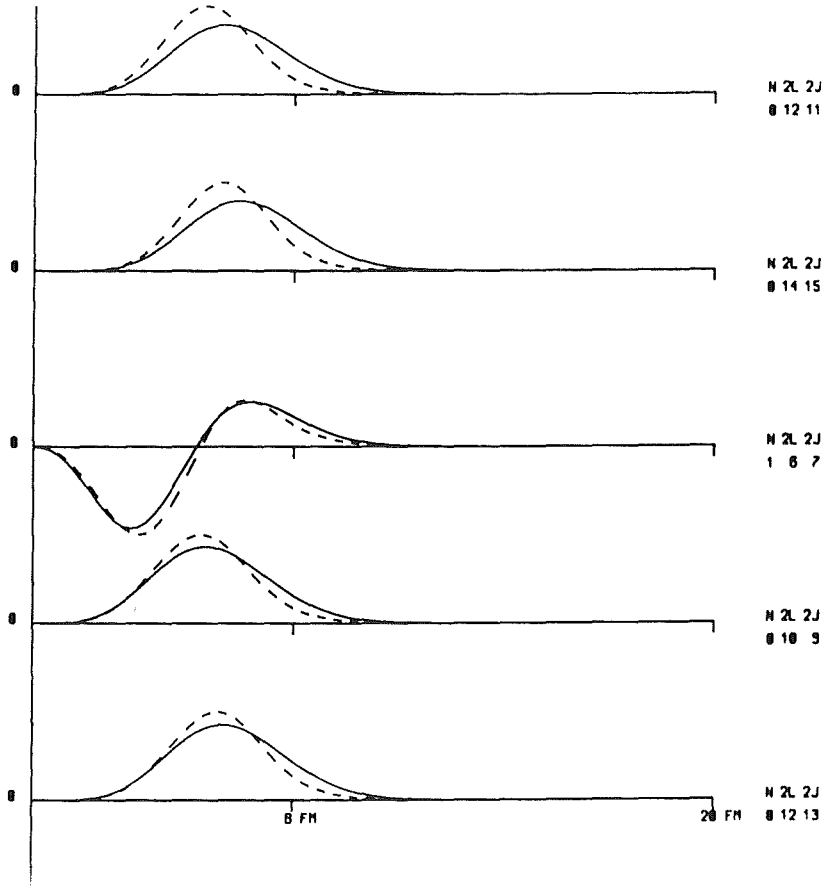


fig. 3.1.1 Proton radial wave functions for different N, L, J. Full line – harmonic oscillator r.w.f., dashed line – Woods-Saxon radial wave functions.

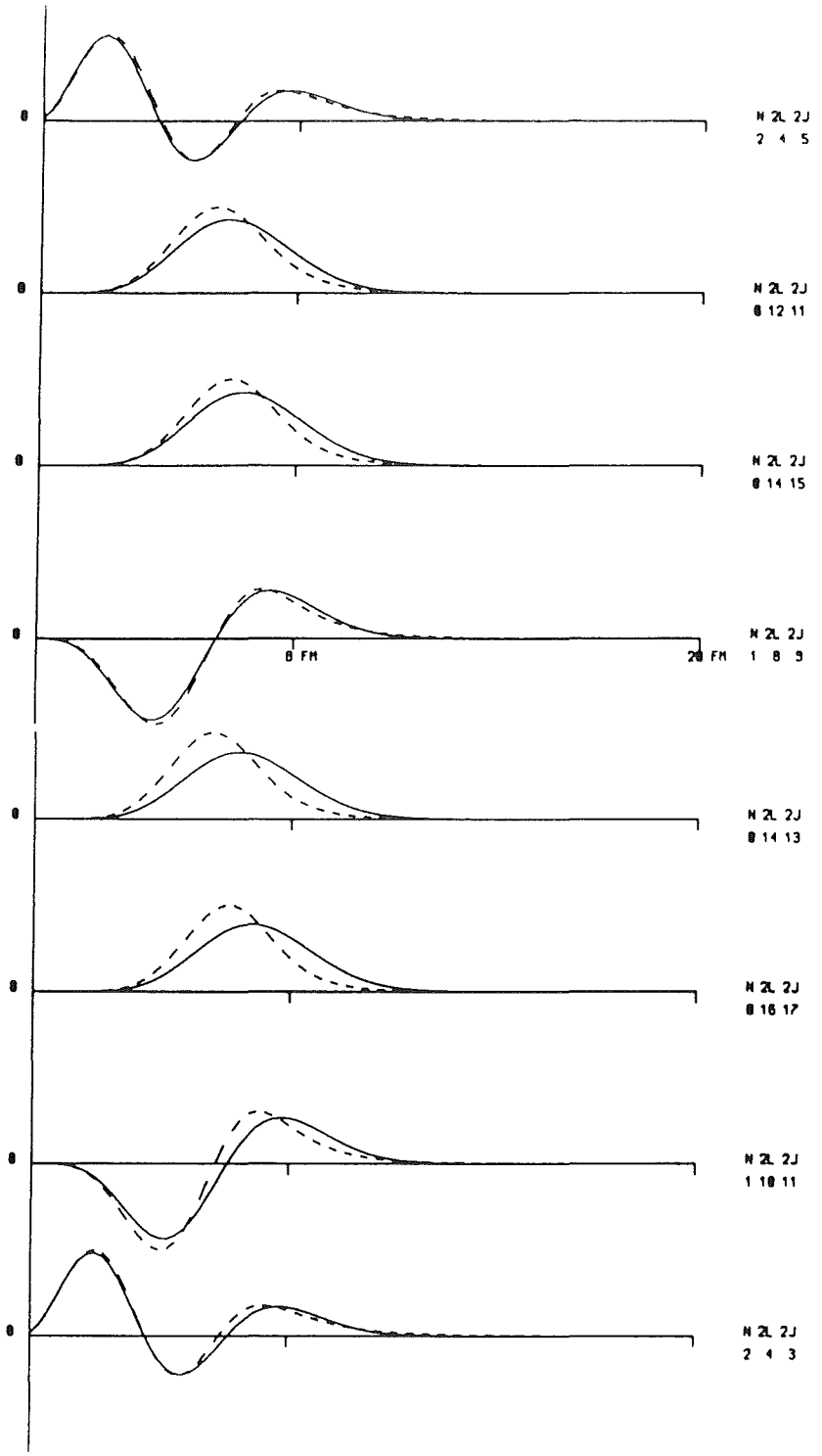


fig. 3.1.2 Proton radial wave functions for different N, L, J. Full line – harmonic oscillator r.w.f., dashed line – Woods-Saxon radial wave functions.

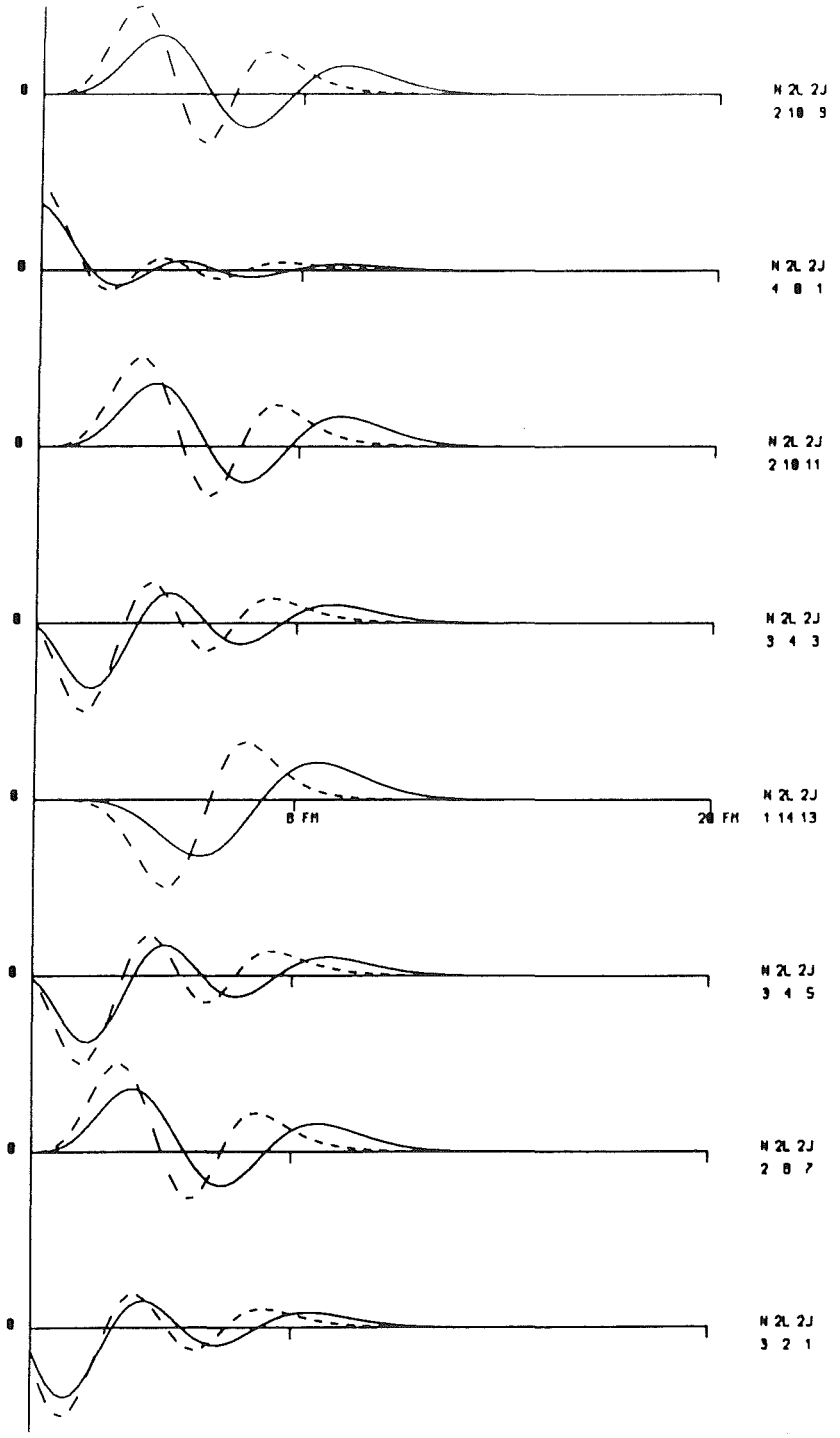


fig. 3.1.3 Neutron radial wave functions for different N, L, J. Full line – harmonic oscillator r.w.f., dashed line – Woods-Saxon radial wave functions.

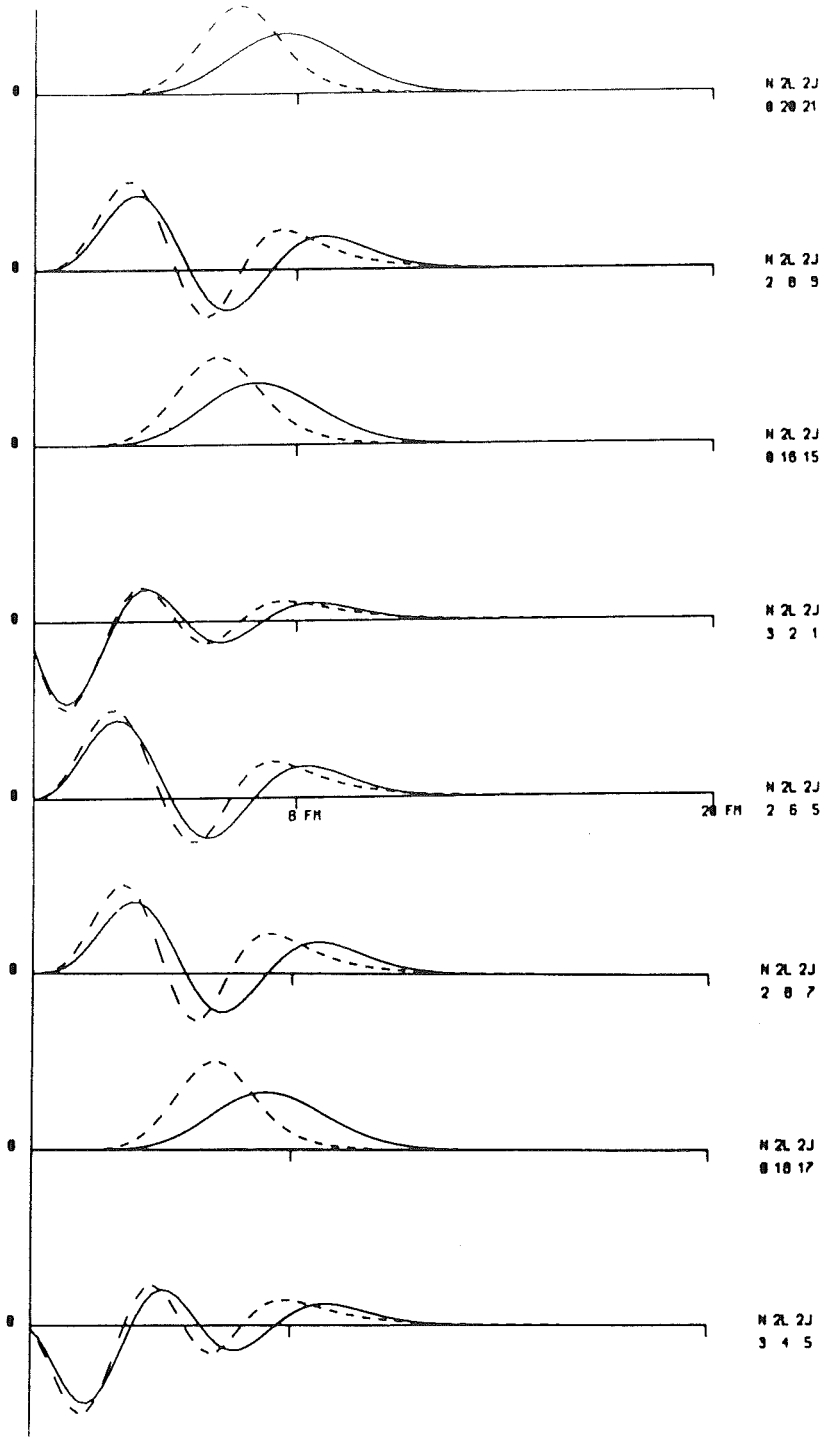


fig. 3.1.4 Neutron radial wave functions for different N, L, J. Full line – harmonic oscillator r.w.f., dashed line – Woods-Saxon radial wave functions.

3.2 Two–particle wave functions

Two-particle wave functions are calculated according to ref. (26), p.113. The residual interaction is supposed to be Surface Delta (SDI). Our earlier calculations with Woods–Saxon potential show that two-particle wave functions are clustered in the surface region of the nucleus and that the clusterisation depends strongly on the number of configurations, ref.(33–36). The strength of SDI is chosen in such a way that the experimental $^{210}Po(g.s.), ^{210}Pb(g.s.)$ and $^{210}Bi(0_1), ^{210}Bi(g.s.)$ energies are correctly reproduced.
The following figure shows clustering of NN pair with harmonic oscillator radial wave functions as a function of R and θ. The same behaviour is found for PP and NP pairs. These two-particle clusters serve afterwards as building blocks for four-particle cluster.

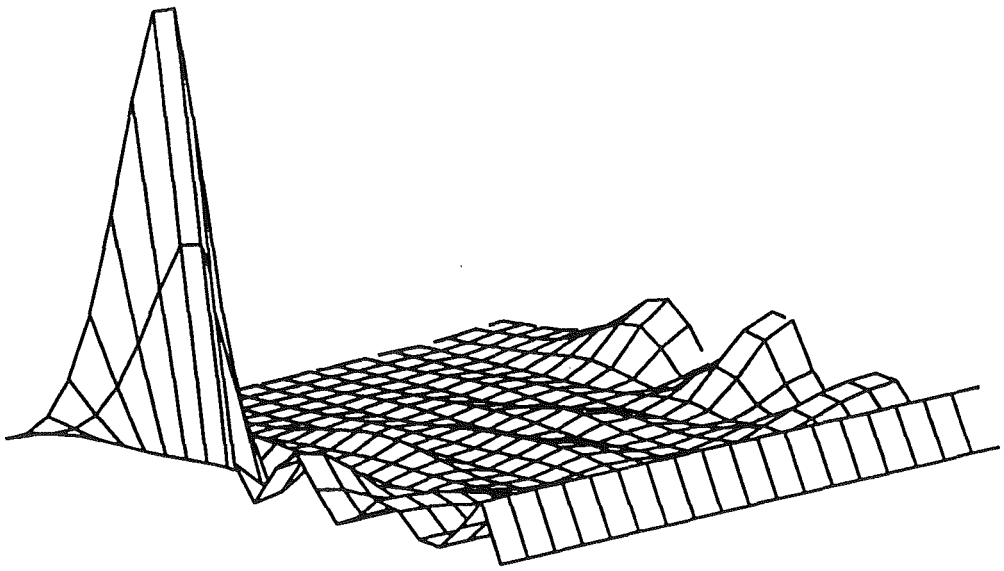


fig. 3.2.1 Two–particle NN cluster

3.3 Cluster wave function

The following figures illustrate how the four-particle cluster wave function depends on the number of configurations:

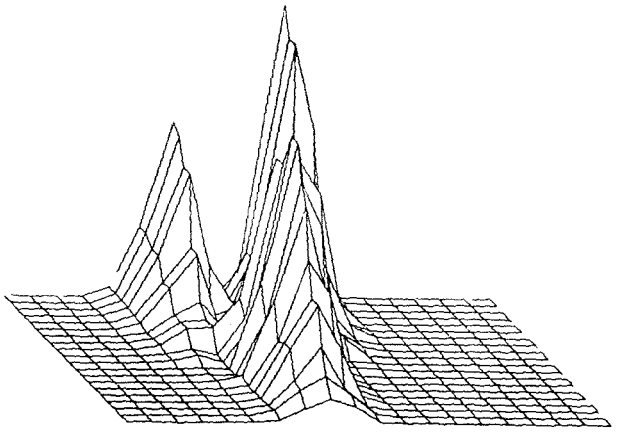


fig. 3.3.1 Probability of α-cluster formation for the main configuration : $(\Psi_C^2)_{max} = 0.13902 \times 10^{-12}$. Coordinates $R_\alpha \in [0,10]fm$, and $r_p = r_n \in [0,4]fm$.

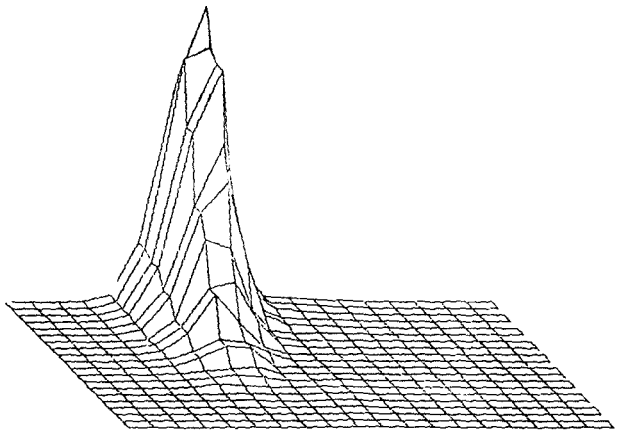


fig. 3.3.2 Ten configurations taken into account. $(\Psi_C^2)_{max} = 0.18629 \times 10^{-9}$. The rest as on fig. 3.3.1.

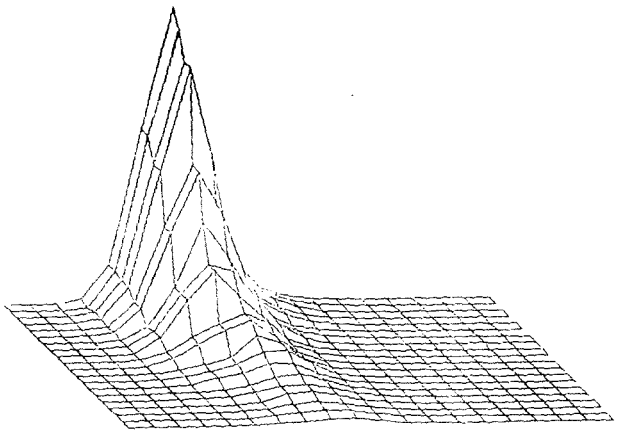


fig. 3.3.3 All 90 configurations taken into account. $(\Psi_C^2)_{max} = 0.35318 \times 10^{-8}$. The rest as on fig. 3.3.1.

The coordinates are given in the following way:

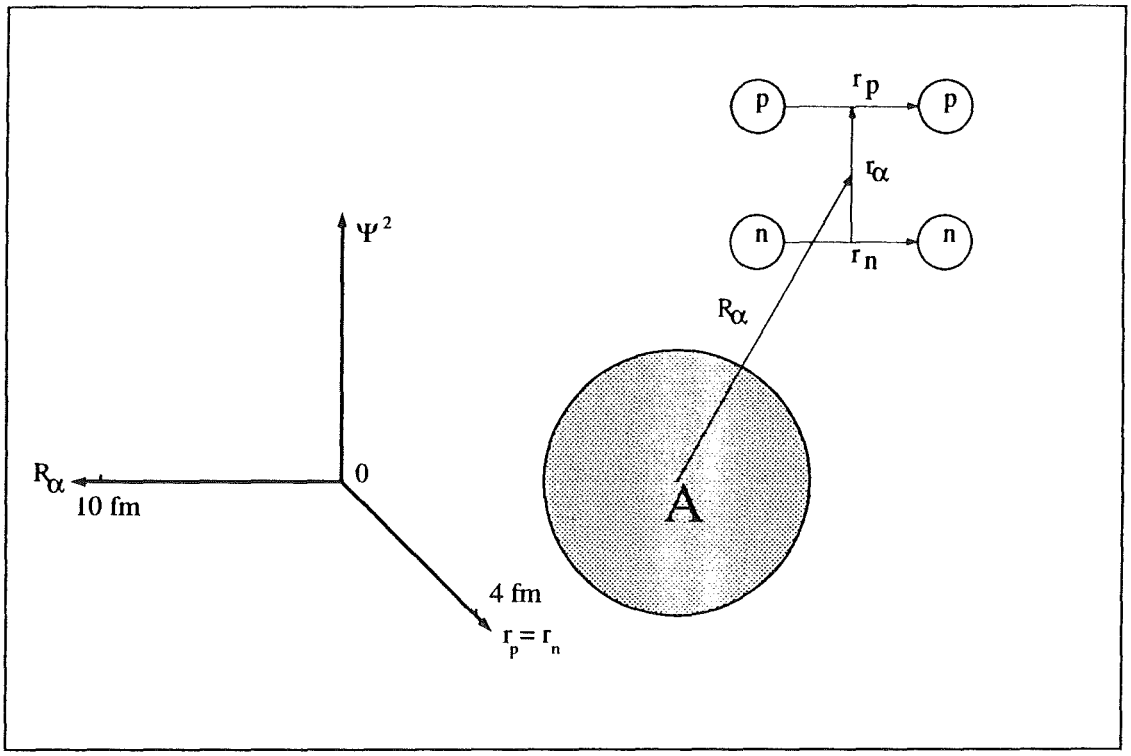


fig. 3.3.4 The coordinates for figs. (3.3.1-3.3.3)

From fig. (3.3.1-3.3.3) one can see what happens when the number of configurations taken into account is increased. The pure configurations only produce two separate peaks corresponding to pair clusters. Increasing of number of configurations makes it possible for protons and neutrons to form an α-cluster. Adding even more configurations increases clustering further. We see that:

$$(\Psi_C^2)_{max}(10conf.) : (\Psi_C^2)_{max}(1conf.) = 1340$$

and

$$(\Psi_C^2)_{max}(90conf.) : (\Psi_C^2)_{max}(1conf.) = 25404.$$

This is how proton-neutron interaction affects α-cluster formation probability:

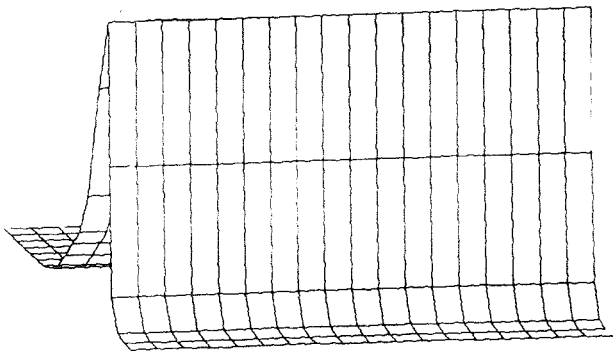


fig. 3.3.5 Probability of α-cluster formation for $X_{PNPN} = 0.$ $(\Psi_C^2)_{max} = 0.15639 \times 10^{-9}$. Coordinates $R_\alpha \in [0, 10] fm$, and $\theta_{pn} \in [0, 180]^0$, with the condition $r_p = r_n = 0.$ 90 conf.

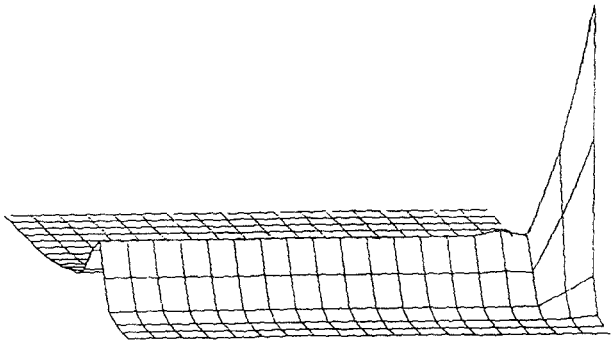


fig. 3.3.6 $X_{PNPN} = 0.3$, $(\Psi_C^2)_{max} = 0.70664 \times 10^{-9}$. The rest as on fig. 3.3.5.

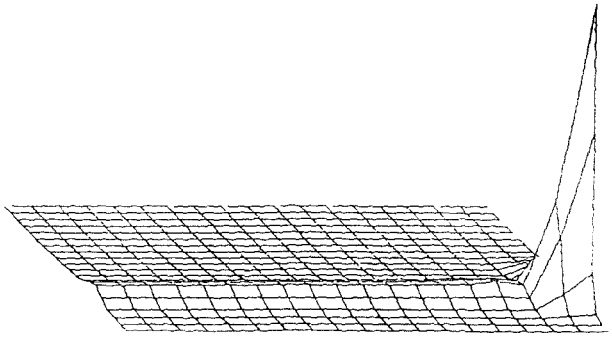


fig. 3.3.7 $X_{PNPN} = 1.0$, $(\Psi_C^2)_{max} = 0.35318 \times 10^{-8}$. The rest as on fig. 3.3.5.

The coordinates are given in the following way:

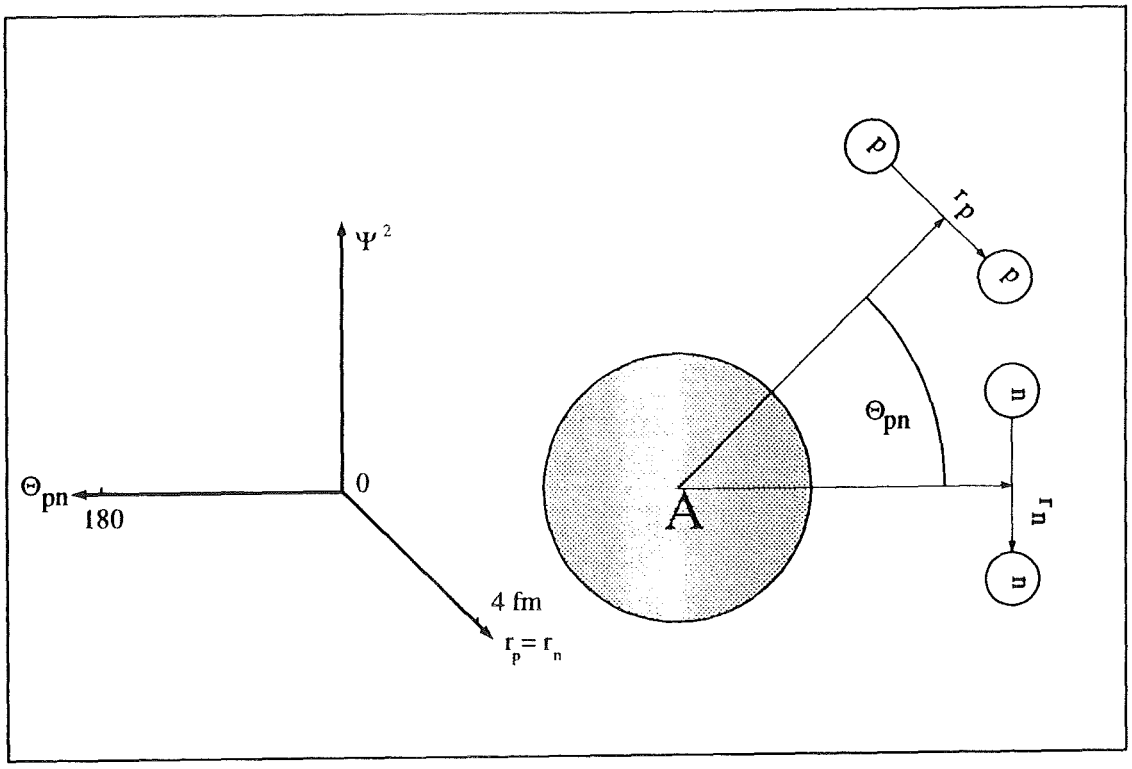


fig. 3.3.8 The coordinates for figs. (3.3.4-3.3.6)

Fig. (3.3.5-3.3.7) show beautifully the importance of proton-neutron interaction. As a consequence of turning off proton-neutron interaction, the probability of α-cluster formation is sphericaly symmetric, and a maximum appears at the surface of the nucleus. After turning on proton-neutron interaction, pronounced maximum emerges for $\theta_{np} = 0$ at the nuclear surface. With strong proton-neutron interaction delta-like shape of probability distribution becomes even more pronounced

$$(\Psi_C^2)_{max}(XPNPN = 0.3) : (\Psi_C^2)_{max}(XPNPN = 0.0) = 4.5$$

and

$$(\Psi_C^2)_{max}(XPNPN = 1.0) : (\Psi_C^2)_{max}(XPNPN = 0.0) = 22.6.$$

Fig. (3.3.1-3.3.7) are the results of the calculations with the Woods–Saxon radial part of the single particle wave functions. All the details of the calculations are presented in ref. (33-36). Using the harmonic oscillator radial wave functions one obtains the same behaviour.

3.4 Formation amplitude and decay half–life

Let me now present the results of the new calculation with harmonic-oscillator radial wave functions. The following two figures illustrate the importance of configuration mixing for the cluster formation amplitude:

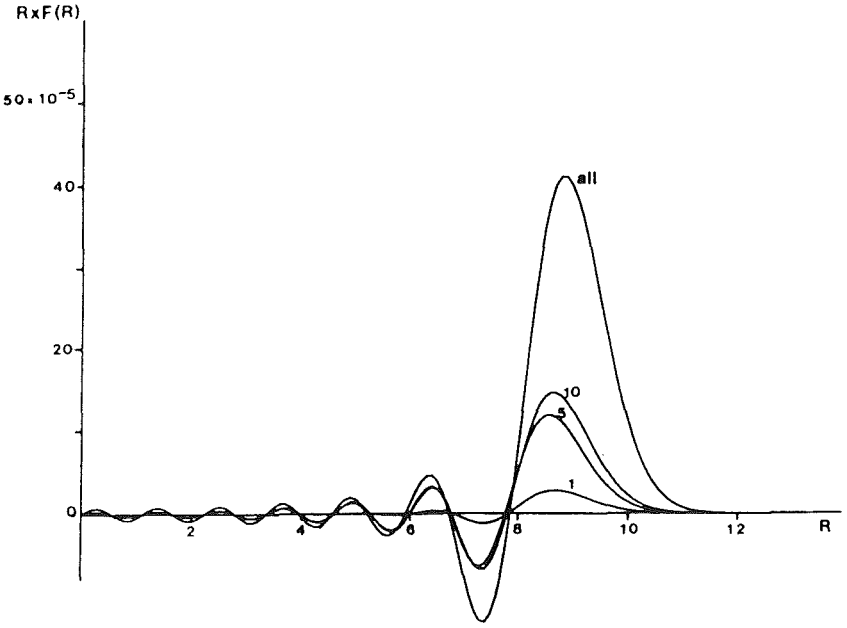


fig. 3.4.1 $F(R) \times R(fm^{-1/2})$ as a function of R_{CM} for the leading PP configuration and for different numbers (on the top of each curve) of NN configurations.

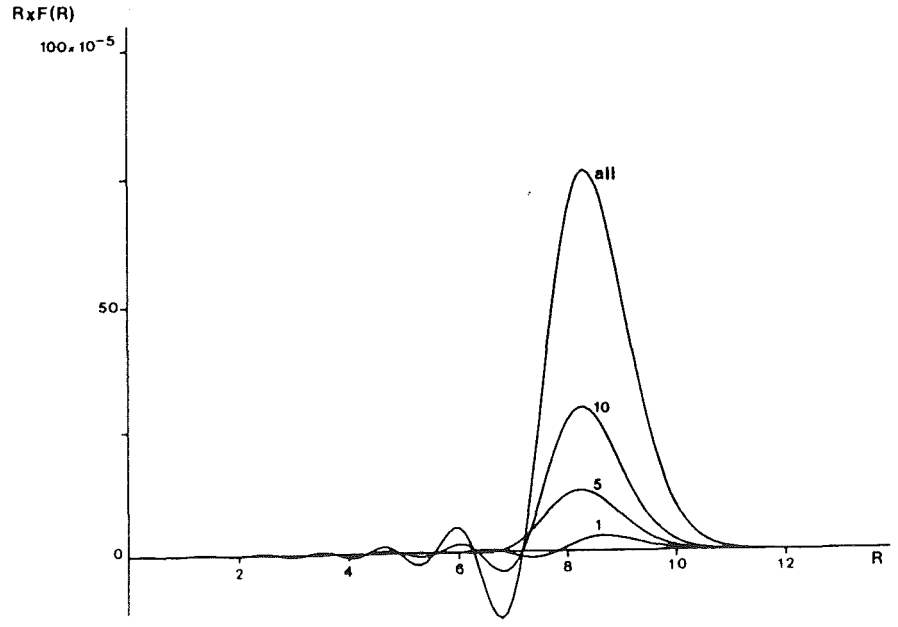


fig. 3.4.2 The same as in previous figure for the leading NN configuration and changing number of PP configurations.

As one would expect from the results for Ψ_C^2, fig. (3.3.1–3.3.3), where cluster wave function shows characteristic behaviour as a function of number of configurations, the formation amplitude itself defined as $F(R) = \int \Phi_{J_C=0}\, Y^*_{LM_L}\, \Phi^*_{J_a=0}\, d\tau$ (19) also strongly increases in the surface of the nucleus and even outside of it. In figs. (3.4.1, 3.4.2) strengths A and B are those obtained from the calculation of the ground state energy of ^{212}Po. A way to see the importance of the neutron-proton interaction in the formation of alpha-particle is to increase the value of B $(XPNPN)$ maintaining the normalisation condition $A^2 + B^2 = 1$.

Here is the evidence for the importance of configuration mixing as well as PN interaction for α-decay calculation. We see that the formation amplitude follows the same pattern as the cluster wave function discussed in chapter 3.3.

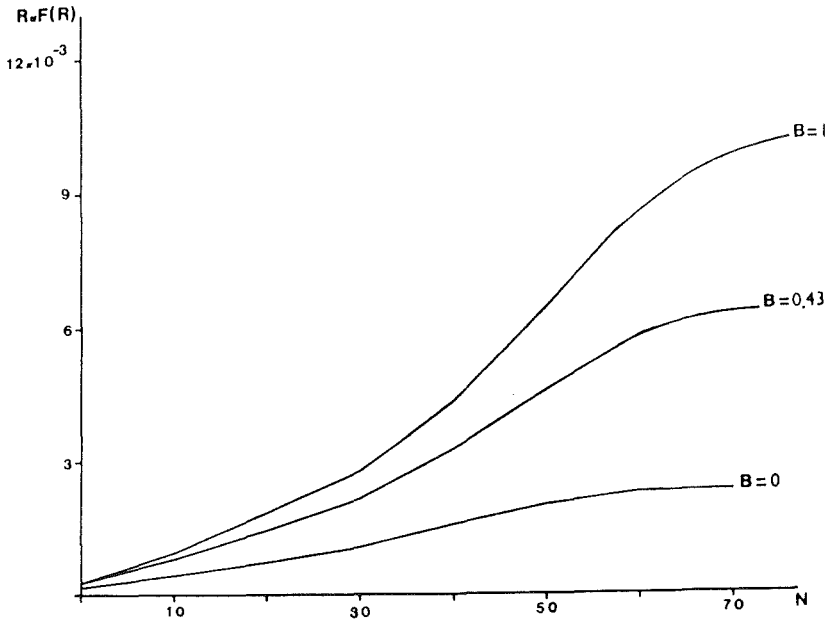


fig. 3.4.3 The dependence of $F(R) \times R(fm^{-1/2})$ on the number of configurations N for $R = 9$ fm and different values of B=X_{PNPN} strength.

We can now analyse the alpha-decay half-life and compare it with the experimental value. The ratio between the theory and experiment depends very much on the strength of neutron-proton contribution (fig. 3.4.4). The experimental values are taken from (ref. 32, 114-117).

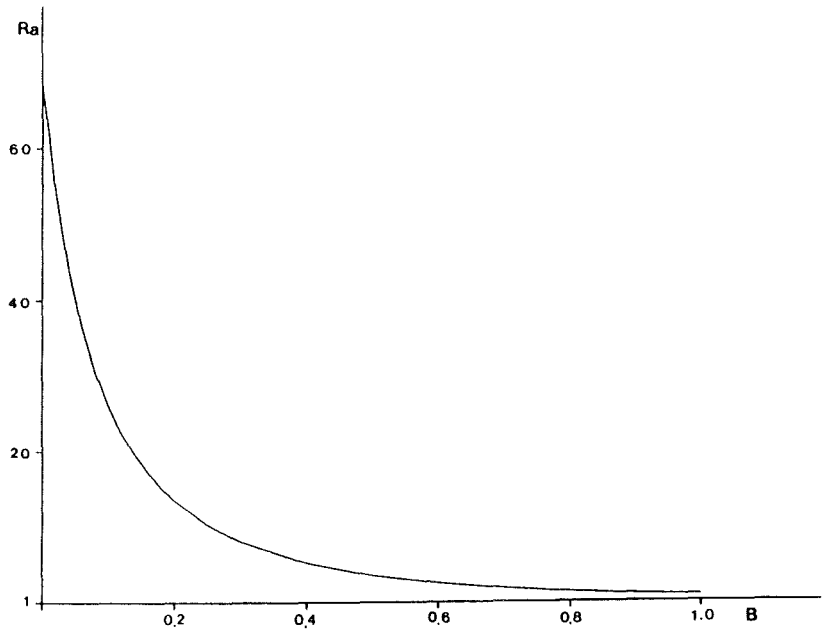


fig. 3.4.4 Ratio Ra=$T_{1/2}(theory)/T_{1/2}(exp)$ as a function of B for R=9 fm. (See Table 3)

The experimental alpha-decay half-lives given by different authors show good agreement with each other (within limits of standard deviation). Schmorak, ref. (104), for example has

$$T_{1/2}^{exp}(^{212}Po) = (0.298 \pm 0.03)\ \mu s.$$

Comparison between theory and experiment for only one(all) configuration(s) taken into account, with different values of B is given in Table 2.

Here is the most interesting result which shows the importance of the NP interaction. The ratio between the experimental and the theoretical α-decay half-life shows a remarkable stability with respect to the radius if PN interaction is turned on. This is a natural consequence of the fact that the neutron-proton interaction increases the alpha-clustering. The more the clustering features are pronounced in the initial state, the better the assumptions of the Gamow theory eq. (21) of α-decay are fulfilled.

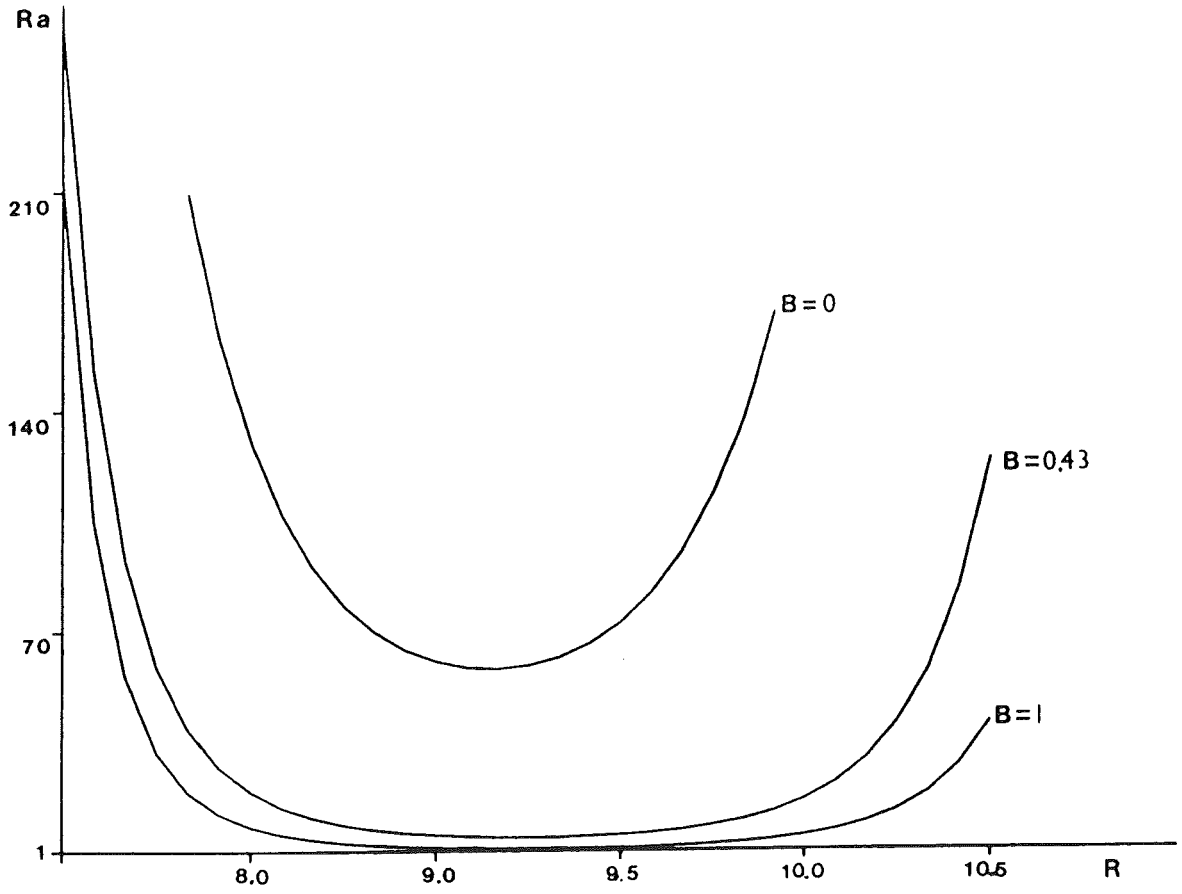


fig. 3.4.5 Ra=$T_{1/2}(theory)/T_{1/2}(exp)$ as function of radius R for different values of B= X_{PNPN} strength. $T_{1/2}(exp) = 0.298\ e^{-6}s$.(See Table 4).

Table 1
$T_{1/2}(th)/T_{1/2}(exp)$ for different strengths $B = X_{NPNP}$.

$R_C(fm)$	$B = 1.0$	$B = 0.43$	$B = 0.0$
8.5	105.84	153.71	539.80
8.6	56.86	93.86	381.89
8.7	32.78	59.96	279.13
8.8	20.12	40.00	210.47
8.9	13.06	27.82	163.50
9.0	8.92	20.13	130.79
9.1	6.39	15.13	107.64
9.2	4.79	11.80	91.06
9.3	3.74	9.54	79.19
9.4	3.04	7.98	70.75
9.5	2.57	6.92	64.92
9.6	2.25	6.19	61.15
9.7	2.05	5.72	59.11
9.8	1.92	5.46	58.65
9.9	1.87	5.37	59.72
10.0	1.88	5.45	62.38
10.1	1.94	5.70	66.84
10.2	2.08	6.14	73.46
10.3	2.29	6.80	82.80
10.4	2.59	7.75	95.69
10.5	3.03	9.08	113.36
10.6	3.64	10.94	137.62
10.7	4.49	13.53	171.31
10.8	5.70	17.20	218.47
10.9	7.44	22.44	285.47
11.0	9.96	30.05	382.27
11.1	13.69	41.29	523.97
11.2	19.31	58.15	735.70
11.3	27.94	84.02	1057.74
11.4	41.46	124.36	1556.46
11.5	63.05	188.74	2346.10
11.6	98.29	293.39	3617.26

Table 2

$T_{1/2}(th)/T_{1/2}(exp)$ for only one configuration taken into account compared with all configurations taken into account, for different strengths $B = X_{NPNP}$.

	$B = 1.0$		$B = 0.43$		$B = 0.0$	
$R_C(fm)$	one	all	one	all	one	all
8.3	1.8E+5	480.1	9.6E+4	478.5	1.6E+5	1.2E+3
8.4	1.1E+5	214.2	7.8E+4	264.2	1.5E+5	789.5
8.5	7.3E+4	105.8	6.5E+4	153.7	1.5E+5	539.8
8.6	5.1E+4	56.9	5.6E+4	93.9	1.5E+5	381.9
8.7	3.8E+4	32.8	5.0E+4	60.0	1.6E+5	279.1
8.8	2.9E+4	20.1	4.5E+4	40.0	1.7E+5	210.5
8.9	2.4E+4	13.1	4.3E+4	27.8	1.9E+5	163.5
9.0	2.1E+4	8.9	4.1E+4	20.1	2.1E+5	130.8
9.1	1.9E+4	6.4	4.1E+4	15.1	2.5E+5	107.6
9.2	1.7E+4	4.8	4.2E+4	11.8	3.1E+5	91.1
9.3	1.7E+4	3.7	4.4E+4	9.5	3.8E+5	79.2
9.4	1.7E+4	3.0	4.7E+4	8.0	4.9E+5	70.8
9.5	1.8E+4	2.6	5.2E+4	6.9	6.4E+5	64.9
9.6	1.9E+4	2.3	5.9E+4	6.2	8.6E+5	61.2
9.7	2.1E+4	2.0	6.9E+4	5.7	1.2E+6	59.1
9.8	2.2E+4	1.9	8.3E+4	5.5	1.7E+6	58.7
9.9	2.9E+4	1.9	1.0E+5	5.4	2.4E+6	59.7
10.0	3.5E+4	1.9	1.3E+5	5.5	3.6E+6	62.4
10.1	4.4E+4	1.9	1.7E+5	5.7	5.5E+6	66.8
10.2	5.7E+4	2.1	2.2E+5	6.1	8.6E+6	73.5
10.3	7.5E+4	2.3	3.0E+5	6.8	1.4E+7	82.8
10.4	1.0E+5	2.6	4.3E+5	7.7	2.3E+7	95.7
10.5	1.4E+5	3.0	6.1E+5	9.1	3.8E+7	113.4
10.6	2.1E+5	3.6	8.9E+5	11.0	6.4E+7	137.6
10.7	3.1E+5	4.5	1.3E+6	13.5	1.1E+8	171.3
10.8	4.6E+5	5.7	2.1E+6	17.2	2.0E+8	218.5
10.9	7.2E+5	7.4	3.3E+6	22.4	3.7E+8	285.5
11.0	1.1E+6	10.0	5.3E+6	30.1	6.8E+8	382.3

Table 3

$F(R) * R$ as function of number of configurations taken into account, corresponding to fig. 3.4.4

N_{CONF}	$B = 1.0$	$B = 0.43$	$B = 0.0$
1	6.8E-04	4.8E-04	2.1E-04
10	1.9E-03	2.4E-03	1.8E-03
20	6.1E-03	6.2E-03	4.0E-03
30	1.1E-02	9.8E-03	5.6E-03
40	1.5E-02	1.3E-02	6.9E-03
50	1.9E-02	1.5E-02	8.0E-03
60	2.4E-02	1.8E-02	8.6E-03
70	2.8E-02	2.0E-02	8.6E-03
80	3.1E-02	2.1E-02	8.6E-03
100	3.3E-02	2.2E-02	8.6E-03

Table 4

(corresponding to fig. 3.4.5).

B	Ra
0.00	69.96
0.02	55.58
0.04	45.25
0.06	37.57
0.08	31.71
0.10	27.14
0.12	23.50
0.14	20.55
0.16	18.14
0.18	16.13
0.20	14.44
0.25	11.24
0.30	9.02
0.40	6.20
0.50	4.56
0.60	3.51
0.70	2.81
0.80	2.33
0.90	1.99
1.00	1.88

CONCLUSIONS

We are describing a process of α-decay of ^{212}Po as a disintegration of initial cluster state into core and a free α-particle.
We demonstrated the necessity of including many configurations to obtain clustering of four nucleons outside the inert core.
It was shown that alpha-cluster appears in the surface region of the nucleus. Once we allowed protons and neutrons to occupy high-lying levels, the argument that they do not interact because of different shells they belong to (N=126, Z=82) has no effect anymore. Therefore we also showed the importance of proton-neutron interaction for cluster formation. Even a small proton-neutron interaction improves clustering and consequently the α–decay width considerably.
Proton-neutron interaction produces cluster which fulfils assumptions of Gamow theory. With increasing proton-neutron interaction the stability of alpha-decay width $\Gamma(R)$ with respect to channel radius is achieved in the large region on the nuclear surface and even beyond it.
The next step would be to calculate some other cases of alpha-decay.
Closely related to α-decay is heavy cluster emission. Besides one-proton and two-protons emission, one can observe many other charged particle radioactivities such as ^{5}He, $^{8,9}Be$, ^{12}B, $^{12,13,14}C$, $^{14,15}N$, $^{16,17,18}O$, $^{21,22}F$, $^{23,24}Ne$, etc. The intensities of these processes are by many orders of magnitude weaker than those of the alpha-decay, of course. It would be interesting to apply our approach to heavy cluster emission as well.
We have still means to improve our alpha-decay calculations. We can hope on the basis of our present results that the better understanding of cluster structure, especially proton–neutron interaction will help to remuve the remaining disagreement with experiment.
We can conclude that our results for ^{212}Po α–decay support the concept of clustering in heavy nuclei.

APPENDIX A

LS-jj coupling coefficients

From (ref. 41) one reads

$$\langle (l_1 l_2) L\ (s_1 s_2) S; J | (l_1 s_1) j_1\ (l_2 s_2) j_2; J \rangle =$$

$$[(2L+1)(2S+1)(2j_1+1)(2j_2+1)]^{1/2} \begin{Bmatrix} l_1 & l_2 & L \\ s_1 & S_2 & S \\ j_1 & j_2 & J \end{Bmatrix}. \tag{76}$$

Since $s_1 = s_2 = \frac{1}{2}$, S may take values 0 or 1. If $S = 0$, that means

$$= (-1)^{l_1 + j_2 + J + 1/2} \sqrt{\frac{(2j_1+1)(2j_2+1)}{2}} \begin{Bmatrix} J & j_1 & j_2 \\ \frac{1}{2} & l_2 & l_1 \end{Bmatrix}. \tag{77}$$

Taking into account that $\lambda_\nu = J_\nu$ $\lambda_\pi = J_\pi$ and $\lambda_\nu = \lambda_\pi$

$$\langle J_\nu\ S_\nu = 0 | j_p j_q \rangle = (-1)^{l_p + j_q + J_\nu + 1/2} \sqrt{\frac{(2j_p+1)(2j_q+1)}{2}} \begin{Bmatrix} J_\nu & j_p & j_q \\ \frac{1}{2} & l_q & l_p \end{Bmatrix} \tag{78}$$

and

$$\langle J_\pi\ S_\pi = 0 | j_r j_s \rangle = (-1)^{l_r + j_s + J_\pi + 1/2} \sqrt{\frac{(2j_r+1)(2j_s+1)}{2}} \begin{Bmatrix} J_\pi & j_r & j_s \\ \frac{1}{2} & l_s & l_r \end{Bmatrix} \tag{79}$$

or, in more special case, when $J_\nu = 0$ and $J_\pi = 0$, $j_p = j_q$ and $l_p = l_q$

$$\langle J_\nu = 0\ S_\nu = 0 | j_p j_q \rangle = (-1)^{l_p + j_q + 1/2} \frac{(2j_p+1)}{\sqrt{2}} \begin{Bmatrix} 0 & j_p & j_q \\ \frac{1}{2} & l_q & l_p \end{Bmatrix} \tag{80}$$

and

$$\langle J_\pi = 0\ S_\pi = 0 | j_r j_s \rangle = (-1)^{l_r + j_s + 1/2} \frac{(2j_r+1)}{\sqrt{2}} \begin{Bmatrix} 0 & j_r & j_s \\ \frac{1}{2} & l_s & l_r \end{Bmatrix}. \tag{81}$$

Using the known relation (ref. 41)

$$\begin{Bmatrix} j_1 & j_2 & j_3 \\ 0 & j_3 & j_2 \end{Bmatrix} = \frac{(-1)^{j_1 + j_2 + j_3}}{\sqrt{(2j_2+1)(2j_3+1)}} \tag{82}$$

that in our case means

$$\begin{Bmatrix} 0\ j_p\ j_p \\ \frac{1}{2}\ l_p\ l_p \end{Bmatrix} = \begin{Bmatrix} \frac{1}{2}\ l_p\ j_p \\ 0\ j_p\ l_p \end{Bmatrix} = \frac{(-1)^{1/2 + l_p + j_p}}{\sqrt{(2l_p+1)(2j_p+1)}} \tag{83}$$

we obtain

$$\langle J_\nu = 0\ S_\nu = 0|j_p j_p\rangle = (-1)^{l_p+j_p+1/2}\ \frac{(2j_p+1)}{\sqrt{2}}\ \frac{(-1)^{1/2+l_p+j_p}}{\sqrt{(2l_p+1)(2j_p+1)}} \tag{84}$$

or

$$\begin{aligned}\langle J_\nu = 0\ S_\nu = 0|j_p j_p\rangle &= \sqrt{\frac{(2j_p+1)}{2(2l_p+1)}} \\ \langle J_\pi = 0\ S_\pi = 0|j_r j_r\rangle &= \sqrt{\frac{(2j_r+1)}{2(2l_r+1)}}.\end{aligned} \tag{85}$$

APPENDIX B

```
PROGRAM SPLEV
C
C PROGRAM TO CALCULATE SINGLE PARTICLE LEVELS FOR RPAPH
C
C
DIMENSION ESP(500), XJ(500), NN(500), nsmall(500), LJ(500), ITZ(500)
1, JJ(500)
11 FORMAT(/, ' NEUTRON STATES', /)
12 FORMAT(/, ' PROTON STATES', /)
13 FORMAT(/, ' KAPPA=', 1F10.4, ' MU=', 1F10.4, /)
15 FORMAT(/, ' MASS=', 1F8.2, ' CHARGE=', 1F8.2, //)
16 FORMAT(/, 'NUMBER OF SINGLE PARTICLE STATES ', I5, /)
14 FORMAT(/, 'I, ITZ, NN, LJ, XJ, ENERGY', /)
18 FORMAT(2X, 4I6, F6.1, 5X, 2F10.5)
OPEN(UNIT=2, NAME='BASE.DAT', STATUS='OLD')
C **********DATA IN
C AM=MASS
C AZ=CHARGE
C NMINN=INITIAL VALUE OF N(PRINCIPAL Q.NUM) FOR NEUTRONS
C NMAXN=FINAL VALUE OF N(PRINCIPAL Q.NUM) FOR NEUTRONS
C NMINP=IDEM FOR PROTONS
C NMAXP=IDEM FOR PROTONS
C FKAPN=KAPPA (NEUTRONS)
C FMUN=MU (NEUTRONS)
C FKAPP (FMUP) =IDEM FOR PROTONS
C THESE VALUES ARE TAKEN FROM NILSSON ET AL (NUCL.PHYS.A 131(1969)1
C AND IN THIS CODE WE HAVE REDEFINED FMU=KAPPA*MU
READ (2, *)AM, AZ, NMINN, NMAXN, NMINP, NMAXP, FKAPN, FMUN, FKAPP, FMUP
C **********END DATA
d OPEN(UNIT=3, NAME='BASE.RES', STATUS='NEW')
C ********UNIT (3) IS FOR INFORMATION (OUTPUT )
C ********UNIT (4) WILL BE USED IN RADIAL.FOR
d OPEN(UNIT=4, NAME='HOSC.BASIS', STATUS='NEW')
OPEN(UNIT=7, NAME='HOSCPROTON.BASIS', STATUS='NEW')
OPEN(UNIT=8, NAME='HOSCNEUTRON.BASIS', STATUS='NEW')
d WRITE(3, 15)AM, AZ
ICOUNT=0
DO 100 ICH=1, 2
```

```
IF(ICH.EQ.1) GO TO 110
FKAPPA=FKAPP
FMU=FMUP*FKAPPA
NMIN1=NMINP+1
NMAX1=NMAXP+1
d WRITE(3, 12)
d WRITE(3, 13)FKAPPA, FMU
FAC=-1.
IFAC=-1
GO TO 112
110 CONTINUE
FKAPPA=FKAPN
FMU=FMUN*FKAPPA
NMIN1=NMINN+1
NMAX1=NMAXN+1
d WRITE(3, 11)
d WRITE(3, 13)FKAPPA, FMU
FAC=1.
IFAC=1
112 CONTINUE
HOMEGA=41.*(1.+ FAC*(AM-2.*AZ)/(3.*AM))/(AM**(1./3.))
d WRITE(3, 2020)HOMEGA
2020 FORMAT(/, 10X, 'OSCILLATOR ENERGY= ', F10.5, /)
DO 200 N1=NMIN1, NMAX1
N=N1-1
FN=N
L0=N-2*(N/2)
DO 210 L=L0, N, 2
ICOUNT=ICOUNT+1
FL=L
FJP=FL+0.5
EP=FN+1.5-FKAPPA*FL-FMU*(FL*(FL+1.)-0.5*FN*(FN+3))
EPMEV=EP*HOMEGA
ITZ(ICOUNT)=IFAC
NN(ICOUNT)=FN
XJ(ICOUNT)=FJP
LJ(ICOUNT)=L
ESP(ICOUNT)=EPMEV
FJP=FL-0.5
IF(FJP.LT.0.0) GO TO 220
ICOUNT=ICOUNT+1
EP=FN+1.5+FKAPPA*(FL+1.)-FMU*(FL*(FL+1.)-0.5*FN*(FN+3))
EPMEV=EP*HOMEGA
```

```
ITZ(ICOUNT)=IFAC
NN(ICOUNT)=FN
LJ(ICOUNT)=L
XJ(ICOUNT)=FJP
ESP(ICOUNT)=EPMEV
220 CONTINUE
210 CONTINUE
200 CONTINUE
100 CONTINUE
NST=ICOUNT
d WRITE(3, 16)NST
d WRITE(3, 14)
DO 300 I=1, NST
AR=ESP(I)
DO 400 J=I, NST
IF(ESP(J).GT.AR) GO TO 400
AR=ESP(J)
TEMP=ESP(J)
ESP(J)=ESP(I)
ESP(I)=TEMP
TEMP=XJ(J)
XJ(J)=XJ(I)
XJ(I)=TEMP
ITEMP=NN(J)
NN(J)=NN(I)
NN(I)=ITEMP
ITEMP=LJ(J)
LJ(J)=LJ(I)
LJ(I)=ITEMP
ITEMP=ITZ(J)
ITZ(J)=ITZ(I)
ITZ(I)=ITEMP
400 CONTINUE
300 CONTINUE
d DO 500 I=1, NST
JJ(I)=2.*XJ(I)+0.000001
ESPO=ESP(I)/HOMEGA
d500 WRITE(3, 18)I, ITZ(I), NN(I), LJ(I), XJ(I), ESPO, ESP(I)
d WRITE(4, *)NST
do 600 i=1, nst
d WRITE(4, *)I, ITZ(I), NN(I), LJ(I), JJ(I), ESP(I)
nsmall(i)=(nn(i)-lj(i))/2
lj(i)=2*lj(i)
```

```
IF(ITZ(I).LT.0)THEN
WRITE(7, *)I, nsmall(I), LJ(I), JJ(I), ESP(I)
ELSE
WRITE(8, *)I, nsmall(i), LJ(I), JJ(I), ESP(I)
ENDIF
600 continue
STOP
END
```

BASE.DAT

```
208. 82. 0 13 0 13 0.0636 0.367 0.0604 0.628
```

```
PROGRAM PPSDI

DIMENSION H(660, 660), NPH(150), LPH(150), JPH(150), EPH(150),
1 NPP(150), LPP(150), JPP(150), EPP(150), NPFON(660), LPFON(660),
2 JPFON(660), OMEGA(660), NHFON(660), LHFON(660), JHFON(660),
3 IL(660), JL(660), OM(660), WFOM(660, 660), AUX(660)
OPEN(UNIT=5, NAME='PPSDI.IN', STATUS='OLD', READONLY)
OPEN(UNIT=7, NAME='PROTSP.DAT', STATUS='OLD', READONLY)
OPEN(UNIT=1, NAME='PPSDI.OUT', STATUS='NEW', CARRIAGECONTROL='LIST')
CALL HELP
READ(5, *)LA, IPI, IT, AT, NSPRI
DO 5 I=1, 660
DO 5 J=1, 660
5 H(J, I)=0.0
WRITE(6, 9600)
I=0
7 I=I+1
READ(7, *, END=8)K, NPH(I), LPH(I), JPH(I), EPH(I)
NPP(I)=NPH(I)
LPP(I)=LPH(I)
JPP(I)=JPH(I)
EPP(I)=EPH(I)
GO TO 7
8 NSPPH=I-1
NSPNP=NSPPH
```

```
WRITE(6, 9100)
KN=0
DO 30 I=1, NSPNP
DO 30 J=I, NSPPH
K=KOUPL(LPH(J), JPH(J), LPP(I), JPP(I), LA, IPI)
IF(K.EQ.0) GOTO 30
KN=KN+1
NPFON(KN)=NPP(I)
LPFON(KN)=LPP(I)
JPFON(KN)=JPP(I)
OMEGA(KN)=EPP(I)+EPH(J)
NHFON(KN)=NPH(J)
LHFON(KN)=LPH(J)
JHFON(KN)=JPH(J)
IL(KN)=I
JL(KN)=J
30 CONTINUE
WRITE(6, 9200)
WRITE(6, *)KN
DO 70 M=2, KN
L=M
TAL=OMEGA(M)
NPTAL=NPFON(M)
LPTAL=LPFON(M)
JPTAL=JPFON(M)
NHTAL=NHFON(M)
LHTAL=LHFON(M)
JHTAL=JHFON(M)
ITAL=IL(M)
JTAL=JL(M)
40 IF(TAL.GE.OMEGA(L-1)) GOTO 50
OMEGA(L)=OMEGA(L-1)
NPFON(L)=NPFON(L-1)
LPFON(L)=LPFON(L-1)
JPFON(L)=JPFON(L-1)
NHFON(L)=NHFON(L-1)
LHFON(L)=LHFON(L-1)
JHFON(L)=JHFON(L-1)
IL(L)=IL(L-1)
JL(L)=JL(L-1)
L=L-1
IF(L.EQ.1) GOTO 60
GOTO 40
```

```
50 OMEGA(L)=TAL
NPFON(L)=NPTAL
LPFON(L)=LPTAL
JPFON(L)=JPTAL
NHFON(L)=NHTAL
LHFON(L)=LHTAL
JHFON(L)=JHTAL
IL(L)=ITAL
JL(L)=JTAL
GOTO 70
60 OMEGA(1)=TAL
NPFON(1)=NPTAL
LPFON(1)=LPTAL
JPFON(1)=JPTAL
NHFON(1)=NHTAL
LHFON(1)=LHTAL
JHFON(1)=JHTAL
IL(1)=ITAL
JL(1)=JTAL
70 CONTINUE
WRITE(6, 9300)
DO 100 I=1, KN
DO 100 J=1, KN
IF(I.EQ.J) H(I, J)=H(I, J)+OMEGA(I)
DR=DELTA(JPFON(I), LPFON(I), JHFON(I), LHFON(I), JPFON(J),
3 LPFON(J), JHFON(J), LHFON(J), LA, IT, AT)
IF(IL(I).EQ.JL(I)) DR=DR/SQRT(2.)
IF(IL(J).EQ.JL(J)) DR=DR/SQRT(2.)
H(I, J)=H(I, J)+DR
100 CONTINUE
WRITE(6, 9400)
ID1=660
CALL EISR1(ID1, KN, H, OM, WFOM, IER, AUX)
IF(IER.NE.0) GOTO 2000
DO 200 I=1, KN
IF(NSPRI.EQ.I)THEN
WRITE(1, *)'ENERGY=', OM(I)
DO J=1, KN
WRITE(1, *)IL(J), JL(J), WFOM(J, I)
ENDDO
ENDIF
DO 180 J=1, KN
AUX(J)=0
```

```
DO 180 K=1, KN
A=0
IF(J.EQ.K) A=OM(I)
180 AUX(J)=AUX(J)+(H(J, K)-A)*WFOM(K, I)
200 CONTINUE
GOTO 2500
2000 WRITE(6, 7500)
2500 CONTINUE
END
```

PPSDI.IN

```
0 1 2 0.028 1
```

```
PROGRAM ALPHA

* THIS PROGRAM CALCULATES ALPHA-DECAY AMPLITUDE
* AS FUCTION OF R,
* (INCLUDING PPNN AND PNPN CONTRIBUTION),
* FROM RMIN TO RMAX IN STEPS OF DR.
* SINGLE PARTICLE R.W.F.ARE HARMONIC OSCILLATOR TYPE.
*
* RADIAL INTEGRATION BY SUBROUTINE XINHO
*
* OUTPUT: FORM-FACTOR (INTEGRAL) FOR ALPHA-DECAY
* (GROUND STATE-GROUND STATE)
* OF 212PO AS FUNCTION OF R,
* ALPHA-DECAY WIDTH FOR 212PO AS FUNCTION OF R.
* initial nucleus, final nucleus, alpha-particle and cluster
* -all of them have spin zero.orbital momentum of alpha-particle
* relative to the daughter L=0.
* pairs of nucleons coupled to zero as well.
*
INCLUDE 'ALPHACOMMON.FOR/LIST'
character*9 datum
character*8 timb
CALL DATE(DATUM)
CALL TIME(TIMB)
C* READS SINGLE-PARTICLE STATES
CALL ONE
write(6, *)' ONE finished.single-particle data read'
C* READS TWO-PARTICLE STATES
CALL TWO
write(6, *)' TWO finished.two-particle data read'
C* CALCULATES Q-COEFFICIENT
CALL QCOEFF
write(6, *)' QCOEFF finished.q coefficient calculated'
OPEN(UNIT=2, NAME='ALPHA_DAT', TYPE='OLD', READONLY)
C OPEN(UNIT=3, NAME='ALPHA_OUT', TYPE='NEW', CARRIAGECONTROL='LIST')
OPEN(UNIT=7, NAME='ALPHAFF_PLT', TYPE='NEW', CARRIAGECONTROL='LIST')
OPEN(UNIT=9, NAME='ALPHAWIDTH_PLT', TYPE='NEW')
C****** READS RADIAL POINTS AT WHICH AMPLITUDE IS CALCULATED *****
READ(2, *)RMIN, RMAX, DR
C****** ANGULAR MOMENTUM OF FOUR-PARTICLE SYSTEM J2P2N=(0)+ ******
C****** ANGULAR MOMENTUM OF PP, NN AND PN PAIRS =0 ***************
100 READ(2, *)XPNPN !READS WEIGHT FOR PPNN contribution
xpnpnsq=xpnpn*xpnpn
```

```
xppnn=dsqrt(1-xpnpnsq) !CALCULATES WEIGHT FOR PNPN contribution
DO 230 I=0, NLIMIT
230 AMP(I)=0.D0
C*
C****** LIMITS FOR THE MOSHINSKY BRACKETS.SEE WRITEMBG PROGRAM ***
C*
OPEN(UNIT=8, NAME='WRITEMBG_DAT', TYPE='OLD', READONLY)
READ(8, *)IX1, IX2, IX3
IOK=0
NMAX=IX2
MAXSNU=IX1
R=RMIN-DR
nradpoint=0
300 R=R+DR
IF(R.GT.RMAX)GO TO 400
nradpoint=nradpoint+1
IF(nradpoint.GT.150)THEN
WRITE(6, 310)
310 FORMAT(' IN ALPHA THE ARRAY RWFA IS OUT OF DIMENSION')
STOP
END IF
C*
C****** FOUR PARTICLE H.O.R.W.F.OF C.M. COORDINATE *********
C*
CALL HOR(NMAX, 0, 4*XNU, R, WF)
DO N=0, NMAX
RWFA(N, nradpoint)=WF(N)
END DO
GO TO 300
400 write(6, *)' hor finished.alpha-particle h.o.radial w.f. calculated.'
CLOSE(UNIT=8)
C*
C***** INTEGRAL OF 2P-2N (PN-PN) RADIAL FUNCTION OVER RELATIVE COORDI-
NATE *****
C*
CALL XINHO(NMAX, 0.5D0)
write(6, *)' XINHO finished.alpha particle radial function integrated.'
C*
C****** AMPLITUDE AS FUNCTION OF R ****************************************
C*
C****** MOSH. BRACK. $\langle NS0NB0;0|N1L1N2L2;0\rangle$

READ FROM CMOSBG.WRI. ***
```

```
*
OPEN(UNIT=8, NAME='MOSBG_WRI', TYPE='OLD',
1 FORM='UNFORMATTED', ACCESS='SEQUENTIAL', READONLY)
240 READ(8, END=2000)NSA, NBA, NBNU, JNU, NBPI, JPI, XMOB
IF (JNU.NE.0)GO TO 1000
AUXP1=XMOB*RINT(NSA)
AUXP2=AUXP1*XPPNN
AUXP22=AUXP1*XPNPN
DO 900 NSNU=0, MAXSNU
QNU=Q(2, NSNU, NBNU)
IF(QNU.GT.1.D9)GO TO 900
AUXP3=AUXP2*QNU
DO 800 NSPI=0, MAXSNU
QPI=Q(1, NSPI, NBPI)
IF(QPI.GT.1.D9)GO TO 800
AUXP=AUXP3*QPI
AMP(NBA)=AMP(NBA)+AUXP
800 CONTINUE
900 CONTINUE
DO 950 NSPN=0, MAXSNU
QPN=Q(3, NSPN, NBNU)
IF(QPN.GT.1D9)GO TO 950
AUXP33=AUXP22*QPN
DO 930 NSNP=0, MAXSNU
QNP=Q(3, NSNP, NBPI)
IF(QNP.GT.1D9)GO TO 930
AUXPP=AUXP33*QNP
AMP(NBA)=AMP(NBA)+AUXPP
930 CONTINUE
950 CONTINUE
1000 CONTINUE
IF((NSA+NBA).EQ.NMAX)IOK=1
GO TO 240
2000 IF(IOK.EQ.0)THEN
WRITE(6, 2100)
2100 FORMAT(' CONDITION MAX(NSA+NBA)=IX2 NOT FULFILLED')
STOP
ENDIF
R=RMIN-DR
write(3, *)' po212 alpha decay width'
WRITE(3, *)' SPIN OF INITIAL NUCLEUS=0'
WRITE(3, *)' SPIN OF FINAL NUCLEUS=0'
WRITE(3, *)' SPIN OF ALPHA PARTICLE=0'
```

```
WRITE(3, *)' PP, NN AND PN PAIRS COUPLED TO ZERO '
WRITE(3, *)
WRITE(3, 2140)XPPNN
WRITE(3, 2141)XPNPN
2140 FORMAT('XPPNN=', F5.2)
2141 FORMAT('XPNPN=', F5.2)
WRITE(3, 2143)NCONF(1)
2143 FORMAT(' number of TWO-PROTON configurations= ', I3)
WRITE(3, 2134)NCONF(2)
2134 FORMAT(' number of TWO-NEUTRON configurations= ', I3)
WRITE(3, 21341)NCONF(3)
21341 FORMAT(' number of PROTON-NEUTRON configurations= ', I3)
WRITE(3, *)
WRITE(3, 2135)NSPS(1)
2135 FORMAT(' number of PROTON SINGLE particle states=', I3)
WRITE(3, 2136)NSPS(2)
2136 FORMAT(' number of NEUTRON SINGLE particle states=', I3)
NUHOHALF=NUHO/2
WRITE(3, 2137)NUHOHALF
2137 FORMAT(' MAXIMAL PRINCIPAL Q.N., NMAX=', I3)
write(3, *)
write(3, *)
write(3, 2145)
2144 FORMAT(' for L=', I3, ' ,  parity=', I3)
2145 format(' R(FM) INTEGRAL INT*R GAMSQ(MEV)')
PI=2.D0*DASIN(1.D0)
AP=PI**3
B=XNUA**2
C=DSQRT(XNUA/AP)
C=DSQRT(C)
CTE=DSQRT(8.D0)*B*C
A=208.
CC=8*983*A/(A+4)
DO 2300 iradpoint=1, nradpoint
R=R+DR
S=0.0
DO 2200 NBA=0, NMAX
2200 S=S+RWFA(NBA, iradpoint)*AMP(NBA)
S=S*CTE
GAM=197.*DSQRT(R/CC)*S
GAMSQ=GAM*GAM
SS=R*S
WRITE(3, 2250)R, S, SS, GAMSQ
```

```
WRITE(7, 2250)R, SS
WRITE(9, 2250)R, GAMSQ
2250 FORMAT(5X, F5.2, 5X, D10.4, 5X, D10.4, 5X, D10.4)
2300 CONTINUE
write(3, *)
write(3, *)datum, ' ', timb
2400 CONTINUE
STOP
END
SUBROUTINE ONE
C
C THIS SUBROUTINE READS SINGLE-PARTICLE DATA (taken from
C Phys.reports 30C(1977)p.305)(OR CALCULATED BY
C SPLEV PROGRAM),
C FIRST FOR PROTONS(M=1), AND THEN FOR NEUTRONS(M=2).
C FORMAT:I, N, 2L, 2J, E
C NSPS(M)-NUMBER OF SINGLE PARTICLE CONFIGURATIONS PROT./NEUT.
C NSP(M, I)-PRINCIPAL QUANTUM NUMBER PROTON/NEUTRON
C LSP(M, I)-2*L FOR PROTON/NEUTRON
C JSP(M, I)-2*J FOR PROTON/NEUTRON
C
INCLUDE 'ALPHACOMMON.FOR'
OPEN (UNIT=1, NAME='PROTSP_DAT', TYPE='OLD', READONLY)
NUHO=0
M=1
10 I=0
100 READ(1, *, END=200)IDUMMY, N, L, J, E
I=I+1
IF(I.GT.N_SINGLE_PART)THEN
WRITE(6, 110)
110 FORMAT(' ONE:MORE THAN N_SINGLE_PART SINGLE-PARTICLE STATES.CHANGE
Z DIMENSIONS')
STOP
END IF
NSP(M, I)=N
LSP(M, I)=L
JSP(M, I)=J
N2L=4*N+L
IF(N2L.GT.NUHO)NUHO=N2L
GO TO 100
200 NSPS(M)=I
M=M+1
IF(M.NE.2) GO TO 300
```

```
CLOSE (UNIT=1)
OPEN(UNIT=1, NAME='NEUTSP_DAT', TYPE='OLD', READONLY)
GO TO 10
300 CONTINUE
CLOSE(UNIT=1)
RETURN
END
SUBROUTINE TWO
C
C THIS SUBROUTINE READS TWO-PARTICLE (PROTON-PROTON,
C NEUTRON-NEUTRON, AND PROTON_NEUTRON) STATES.
C Two-particle states are calculated by PPSDI, NNSDI, AND PNSDI programs
C by diagonalysing surface-delta interaction between two particles
C see: Glaudemans, Brussard textbook.
C Single-particle data are taken from Phys.Reports 30C(1977)305,
c or calculated by SPLEV program.
C M=1 (PROTON PAIRS), M=2 (NEUTRON PAIRS), M=3 (PROTON-NEUTRON PAIRS).
C FORMAT:
C I1, I2, X
C I1=ITWO(M, KONF):POINTER TO THE SINGLE PARTICLE STATE
C (NSP, LSP, JSP) FOR THE FIRST PARTICLE.
C I2=JTWO(M, KONF):POINTER TO THE SINGLE PARTICLE STATE
C (NSP, LSP, JSP) FOR THE SECOND PARTICLE.
C
C KONF:DENOTES TWO-PARTICLE CONFIGURATIONS
C (I.E. PAIRS OF SINGLE PARTICLE STATES WITH CORRESPONDING
C WEIGHTS X) .
C YTWO(M, KONF):WEIGHT X OF TWO PARTICLE CONFIGURATION
C MULTIPLIED BY CONSTANT.
C YTWO=X*(2**1/2)*PHASE*ÄJP*JQÅ**1/2*SIXJ(JP, LP, 1/2, LQ, JQ, JNU)
C X=WEIGHT OF GIVEN (I1, I2) TWO-PARTICLE CONFIGURATION
C X COMES FROM THE EXPRESSION FOR THE TWO-PARTICLE W.F.:
C PSI(R1, R2;ALPHANU)=(SUM OVER P.LE.Q)ÄX(P, Q;ALPHANU)*
C ÄPHIP(R1)*PHIQ(R2)ÅÅ
C NCONF(M):NUMBER OF TWO-PARTICLE CONFIGURATIONS
C
C
INCLUDE 'ALPHACOMMON.FOR'
CHARACTER*40 TWONAME
CALL HELP
M=1
10 GO TO (20, 30, 40, 2000)M
20 TWONAME='PP_DAT'
```

```
GO TO 50
30 TWONAME='NN_DAT'
GO TO 50
40 TWONAME='PN_DAT'
50 OPEN(UNIT=1, NAME=TWONAME, TYPE='OLD', READONLY)
XNORM=0.
KONF=0
300 READ(1, *, END=500)I1, I2, X
IF(I1.LT.0) GO TO 500
KONF=KONF+1
IF(KONF.GT.N_TWO_PART)STOP 'N_TWO_PART TOO SMALL'
IF(M.EQ.3)THEN
MFIRST=1
MSECOND=2
ELSE
MFIRST=M
MSECOND=M
ENDIF
ITWO(M, KONF)=I1
JTWO(M, KONF)=I2
NP=NSP(MFIRST, I1)
LP=LSP(MFIRST, I1)
JP=JSP(MFIRST, I1)
NQ=NSP(MSECOND, I2)
LQ=LSP(MSECOND, I2)
JQ=JSP(MSECOND, I2)
JNU=0
IF(LP.NE.LQ)GOTO 300
IF(JP.NE.JQ)GOTO 300
XNORM=XNORM+X*X !norm from sum(x**2)=1
a=dfloat(2*(lp+1))
b=dfloat(jp+1)
c=b/a
c=dsqrt(c)
SYMFACT=DSQRT(2.D0) !SYMMETRIZATION FACTOR
IF(NP.EQ.NQ)SYMFACT=1.
f=1.
lphalf=lp/2
if(lphalf.ne.2*(lphalf/2))f=-1.
YTWO(M, KONF)=X*c*SYMFACT*f !YTWO IS Y(PQ;ALPHANU), NOTES!
GO TO 300
500 NCONF(M)=KONF
XNORM=DSQRT(XNORM)
```

```
write(6, 599)xnorm
WRITE(6, *)TWONAME
599 format(' TWO: xnorm='f5.3)
DO I=1, KONF
YTWO(M, I)=YTWO(M, I)/XNORM
END DO
CLOSE(UNIT=1)
M=M+1
GO TO 10
2000 RETURN
END
SUBROUTINE QCOEFF
C*
C* THIS SUBROUTINE CALCULATES THE COEFFICIENTS Q
C* FOR PROTON PAIRS (M=1), NEUTRON PAIRS (M=2),
C* AND PROTON-NEUTRON PAIRS(M=3)(SEE THE NOTES).
C* Q=(SUM)( YTWO * Z4 )
C* YTWO=TWO-PARTICLE WEIGHT X MULTIPLIED BY CONSTANT.
C* Z4= MOSH.BRACKET*INTEGRALÄRNL(R)*EXP(-0.25*XNU*R**2)*R**2*DRÅ
C* MOSHINSKY BRACKETS ARE READ FROM MOSBR.WRI.
C* RADIAL INTEGRAL THAT ENTERS Z4 IS CALCULATED IN
C* SUBROUTINE XINHO(N, 1/4).
C* IN NUHO MAX Ä2(NP+NQ)+LP+LQÅ.
C*
C* TO EXECUTE LINES WITH D-COMMENT MAKE: FOR/D_
C*
C*
INCLUDE 'ALPHACOMMON.FOR'
NUSMAX=NUHO
DO 50 I3=0, NLIMIT
DO 50 I2=0, NLIMIT
DO 50 I1=1, 3
50 Q(I1, I2, I3)=1.d20
IF(NUSMAX.GT.NLIMIT)THEN
WRITE(6, 90)NUHO
90 FORMAT('QCOF: CHANGE DIMENSION, NUHO.GT.NLIMIT, NUHO=')
STOP
END IF
C*
C****** RADIAL INTEGRALS OF R(NSNU, 0), NEUTRON-PAIR RELATIVE COORDI-
NATE,
C****** R(NSPI, 0), PROTON-PAIR RELATIVE COORDINATE, AND
C****** R(NSPN, 0)PROTON-NEUTRON RELATIVE COORDINATE
```

```
C*
CALL XINHO(NUSMAX, 0.25D0)
C*
C****** range of ⟨NS, 0, NB, LA; LA|N1, L1, N2, L2; LA⟩
C*
OPEN(UNIT=8, NAME='WRITEM_DAT', TYPE='OLD', READONLY)
C*
C* LAWRM IS MAXIMUM LAMBDA
C* N1.LE.NPWRM, N2.LE.NPRWM
C* 2N1+L1.LE.LPWRM, 2N2+L2.LE.LPWRM
C* NBPQR.GE.2(N1+N2)+L1+L2
C* FOR THE DETAILS SEE THE PROGRAM WRITEM.
C*
READ(8, *)LAWRM, NPWRM, LPWRM, NBPQWR
CLOSE(UNIT=8)
C*
C* READS ⟨NS, 0, NB, LA; LA|N1, L1, N2, L2; LA⟩
C* LAU8 IS LAMBDA FROM UNIT=8, AND LIKE.
C*
OPEN(UNIT=8, NAME='MOSBR_WRI', TYPE='OLD', READONLY,
Z FORM='UNFORMATTED', ACCESS='SEQUENTIAL')
400 READ(8, END=2000)LAU8
DO 430 M=1, 3
JNU=0
IF(JNU.GT.LAWRM)GO TO 460
IF(JNU.EQ.LAU8)GO TO 470
420 CONTINUE
430 CONTINUE
440 READ(8)NSU8, NBU8, N1U8, L1U8, N2U8, L2U8, AU8
IF(NSU8.LT.0)GO TO 400
GO TO 440
460 WRITE(6, 461)JNU, LAWRM
461 FORMAT(' IN QCOF JNU, LAWRM=', 2I3)
STOP
470 CONTINUE
500 READ(8)NSU8, NBU8, N1U8, L1U8, N2U8, L2U8, AU8
IF(NSU8.LT.0)GO TO 400
DO 1800 M=1, 3
NSM=NSU8 ! NSM MEANS N SMALL
IF(JNU.NE.LAU8)GO TO 1700
KONF=NCONF(M)
DO 1600 K=1, KONF
I1=ITWO(M, K)
```

```
I2=JTWO(M, K)
IF(M.EQ.3)THEN
MFIRST=1
MSECOND=2
ELSE
MFIRST=M
MSECOND=M
ENDIF
NP=NSP(MFIRST, I1)
LP=LSP(MFIRST, I1)/2
NQ=NSP(MSECOND, I2)
LQ=LSP(MSECOND, I2)/2
IS=2*(NP+NQ)+LP+LQ
IF(NP.GT.NPWRM.OR.LP.GT.LPWRM.OR.IS.GT.NBPQWR)THEN
WRITE(6, 550)NP, NPWRM, LP, LPWRM, IS, NBPQWR, K, N2
550 FORMAT(' IN QCOF CONDITIONS NOT SATISFIED', 8I3)
STOP
END IF
NBG=NBU8 ! NBG MEANS N BIG
IF(NP.NE.N1U8.OR.LP.NE.L1U8.OR.NQ.NE.N2U8.OR.LQ.NE.L2U8)
Z GO TO 1600
XM=AU8
Z4=XM*RINT(NSM) ! Z4 OF THE NOTES
IF(Q(M, NSM, NBG).GT.1.D19)Q(M, NSM, NBG)=0.D0
Q(M, NSM, NBG)=Q(M, NSM, NBG)+YTWO(M, K)*Z4
1600 CONTINUE
1700 CONTINUE
1800 CONTINUE
GO TO 500
2000 CONTINUE
CLOSE(UNIT=8)
RETURN
END
SUBROUTINE XINHO(NUSMAX, CASE)
INCLUDE 'ALPHACOMMON.FOR'
C*
C****** THIS SUBROUTINE CALCULATES THE RADIAL INTEGRAL
C****** INT(R**2*EXP((-XNU*CASE)*R**2)RNL(R), FROM N=0 TO N=NUSMAX.
C*
C* NUSMAX=NUHO
C* NUHO=MAXÄ2(NP+NQ)+LP+LQÅ
C* HARMONIC OSCILLATOR RADIAL WAVE FUNCTIONS
C* ARE CALCULATED IN HOR
```

```
C* INTEGRATION METHOD:GAUSS(ABRAMOWITZ, P887)
C*
DIMENSION XG(4), WG(4)
DO 60 I1=0, NLIMIT
60 RINT(I1)=0.
XG(1)=-0.861136311594053
XG(2)=-0.339981043584856
XG(3)=-XG(2)
XG(4)=-XG(1)
WG(1)=0.347854845137454
WG(2)=0.652145154862546
WG(3)=WG(2)
WG(4)=WG(1)
RMAX=20.
NSTEPS=25
DR=RMAX/NSTEPS
NMAX=NUSMAX
B=0.
100 CONTINUE
A=B
B=B+DR
IF(B.GT.RMAX)GO TO 350
DO 300 IG=1, 4
YI=0.5*(DR*XG(IG)+A+B)
YISQ=YI*YI
AA=CASE*XNUA*YISQ
L=0
CALL HOR(NMAX, L, XNU, YI, WF) !2-PARTICLE H.O.R.W.F.
DO 200 N=0, NMAX
DEX=DEXP(-AA)
FUNC=YISQ*WF(N)*DEX
RINT(N)=RINT(N)+0.5*DR*WG(IG)*FUNC
200 CONTINUE
300 CONTINUE
GO TO 100
350 CONTINUE
RETURN
END
SUBROUTINE HOR(NMAX, L, XMU, R, HOWF)
C*
C* THIS SUBROUTINE CALCULATES ALL HARMONIC OSCILLATOR RADIAL
C* WAVE-FUNCTIONS FOR n.LE.NMAX,  WHERE n IS THE PRINCIPAL
C* QUANTUM NUMBER (.GE.0),  L IS THE ORBITAL ANGULAR MOMENTUM,
```

```
C* XMU IS SIZE PARAMETER,  R IS DISTANCE (IN FM) AND THE
C* RESULTS ARE ACCUMULATED IN THE ARRAY HOWF.
C* FORMULA FROM M.MOSHINSKY, NUCL.PHYS.13(1959)105.
C*
INCLUDE 'ALPHACOMMON.FOR'
DIMENSION FDF(0:100), HOWF(0:50), P(0:50)
COMMON/HEL/FN(0:100)
XNORM=XMU*DSQRT(XMU)
XNORM=DSQRT(XNORM)
C2=DLOG(2.D0)
CPI=2.*DASIN(1.D0) !ARC SINUS
CPI=DLOG(CPI)*0.5
FDF(1)=0.
DO I=3, 99, 2
X=DFLOAT(I)
FDF(I)=FDF(I-2)+DLOG(X) !FDF(I)=LN(I)!!
END DO
RF=DSQRT(XMU)*R
XX=RF*RF
IF(R.EQ.0.)THEN
REL=0.D0
ELSE
REL=RF**L
ENDIF
ER2=DEXP(-0.5*XX)
C CALL PLX(NMAX, L, XX, P) !PLX CALCULATES LAGUERRE POL.
P(0)=ER2
A=L+0.5
P(1)=(-XX+A+1)*ER2
DO N=0, NMAX
IF(N.GE.2) THEN
C=(DFLOAT(2*N-1)+A-XX)/DFLOAT(N)
D=(DFLOAT(N-1)+A)/DFLOAT(N)
P(N)=C*P(N-1)-D*P(N-2)
ENDIF
J=2*(N+L)+1
SQ=0.5*(C2*DFLOAT(N+L+2)+FN(N)-FDF(J)-CPI)
SQ=DEXP(SQ)
C phase factor introduced to agree with Woods-Saxon
IPHASE=(-1)**N
c IPHASE=1.
HOWF(N)=SQ*REL*P(N)*XNORM*IPHASE
END DO
```

```
RETURN
END
```

REFERENCES

1. B.J.Allen, J.H.Gibbons, and R.L.Macklin, " Nucleosynthesis and Neutron-capture Cross Sections", *Advances in Nuclear Physics* , Vol.4(1971), p.205, Ed. M.Baranger and E.Vogt

2. Akito Arima, " Clustering Aspects of Nuclear Structure and Nuclear Reactions", *Clustering Aspects of Nuclear Structure and Nuclear Reactions*, Winnipeg 1978.

3. Akito Arima, " Shell Model, Quartets and Clusters in Light Nuclei", *Dynamic Structure of Nuclear States* , Proc. of 1971 Mont Tremblant Int. Summer School, 1972. Ed. D.J.Rowe, L.E.H.Trainor, S.S.M.Wong, T.W.Donnely .

4. Akito Arima, V.Gillet, J.Ginocchio, " Energies of Quartet Structures in Even-even N=Z Nuclei", *Phys.Lett.* **25**(1970)1043

5. Akito Arima, H.Horiuchi, K.Kubodera and N.Takigawa, "Clustering in Light Nuclei", *Advances in Nuclear Physics* , Vol.**5**.(1972)345, Ed. M.Baranger and E.Vogt

6. Akito Arima and S.Yoshida, " Analysis of Alpha-decay Widths of Light 4n Nuclei", *Nucl.Phys.* **A219** (1974)475

7. Akito Arima and V.Gillet, *Ann.Phys.* (N.Y.)**66** (1971)117

8. Akito Arima and Tokuo Terasawa, "Spin-Orbit Splitting and Tensor Force.II", *Progr.Theor.Phys.* **23** (1960)115

9. J.Bang, C.Dasso, F.A.Gareev, M.Igarashi, B.S.Nilsson, *Nucl.Phys.* **A264**(1976)157

10. M.Baranger and K.T.R.Davies, "Oscillator Brackets for Hartree-Fock Calculations", *Nucl.Phys.* **79** (1966)403

11. C.A.Barnes, "Nucleosynthesis by Charged-Particle Reactions", *Advances in Nuclear Physics* , Vol.**4** (1971), p.133, Ed. M.Baranger and E.Vogt

12. A.R.Barnett, D.H.Feng, J.W.Steed and L.J.B.Goldfarb, "Coulomb Wave Functions for all Real Eta and Rho", *Computer Phys.Commun.* 8(1974)377

13. M.K.Basu, "Alpha-Cluster Structure of Light Self-Conjugate 4n Nuclei in their Ground States", *Phys.Rev* .**C6** (1972)476

14. M.K.Basu and D.Banerjee, "The Role of Alpha-Cluster in Natural Alpha Decay", *Clustering Aspects of Nuclear Structure and Nuclear Reactions* (Winnipeg, 1978)

15. B.F.Bayman and A.Kallio, "Relative-Angular-Momentum-Zero Part of Two-Nucleon Wave Functions", *Phys.Rev.* **156**(1967)1121

16. F.D.Beccheti, "Alpha Clustering Studied with the $(^6Li, d)$ and $(d, ^6Li)$ Reactions", *Clustering Aspects of Nuclear Structure and Nuclear Reactions* (Winnipeg, 1978)

17. G.F.Bertsch, *The Practitioner's Shell Model* , North-Holland Publ. Comp., 1972

18. J.M.Blatt and V.F.Weisskopf, *Theoretical Nuclear Physics* , Springer-Verlag, 1979

19. J.Blomqvist and S.Wahlborn, "Shell Model Calculations in the Lead Region with a Diffuse Nuclear Potential", *Arkiv för Fysik***16**(1960)545

20. A.Bohr and B.R.Mottelson, "Two-Nucleon Transfer Modes. Pair Vibrations", in :*Nuclear Structure* , Vol.II, p.387, W.A.Benjamin, Inc., 1975

21. R.Bonetti, L.Colombo and K.I.Kubo, "Inelastic Alpha-scattering to the Continuum: A Probe of Alpha-clustering in Nuclei", *Nucl.Phys* .**A420** (1984)109

22. P.F.Bortignon, R.A.Broglia, D.R.Bes and R.J.Liotta, "Nuclear Field Theory", *Phys.Rep.* **30C** (1977)305

23. D.M.Brink and J.J.Castro, "Alpha Clustering Effects in Nuclear Matter", *Nucl.Phys.***A216**(1973)109

23.1 T.A.Brody and M.Moshinsky, *Tables of Transformation Brackets*, Monografias del Instituto de Fisica, Mexico, 1960

24. R.A.Broglia and P.F.Bortignon, "Alpha-vibrations", *Phys.Lett.* **65B**(1976)221

25. B.Bremond and J.G.Valatin, "A Method to Describe Pairing Correlations of Protons and Neutrons", *Nucl.Phys.* **41**(1963)640

26. P.J.Brussard and P.W.M.Glaudemans, *Shell-Model Applications in Nuclear Spectroscopy* , North Holland Publ.Co., 1977

27. F.Carstoiu and O.Dumitrescu, "On the Pauli Corrections in the Sub-Coulomb Transfer Reactions", *Prepr.* $IC/81/240$

28. F.Catara, A.Insolia and U.Lombardo, "Pairing and Quarteting Correlations in Nuclei", *Nucl.Phys.* **A261**(1976)282

29. F.Catara, A.Insolia, E.Maglione and A.Vitturi, "Relation Between Pairing Correlations and Two-Particle Space Correlations", *Phys.Rev.* **C 29**(1984)1091

30. H.Daley and F.Iachello, "Alpha-Clustering in Heavy Nuclei", *Phys.Lett.***131B**(1983)281

31. M.Danos, "SU4 in Nuclear Structure", *Clustering Aspects of Nuclear Structure and Nuclear Reactions*, Winnipeg 1978.

32. W.G.Davies, R.M.De Vries, G.C.Ball, J.S.Forster, W.McLatchie, D.Shapira, J.Toke and R.E.Warner, "A Comparison of Alpha-Transfer and Alpha-Decay in the Lead Region", *Nucl.Phys.* **A269**(1976)477

33. G.Dodig-Crnković, F.A.Janouch and R.J.Liotta, "Alpha–Clustering and Absolute Alpha–Decay Widths in Spherical Nuclei", *Phys.Lett.***139B**(1984)143

34. G.Dodig-Crnković, F.A.Janouch and R.J.Liotta, "Absolute Alpha–Decay Widths in Po Isotopes", *Proc.Int.Symp. on In-beam Spectroscopy, Debrecen* , 1984

35. G.Dodig-Crnković, F.A.Janouch, R.J.Liotta and L.J.Sibanda, "Absolute Alpha-Decay Rates in Po Isotopes", *Nucl.Phys.* **A444**(1985)419

36. G.Dodig-Crnković, F.A.Janouch and R.J.Liotta, "Absolute Alpha-Decay Widths", *Proc. Yugoslav Meeting of Nuclear and Particle Physicists, Portoroz* , 1985

37. G.Dodig-Crnković, F.A.Janouch, R.J.Liotta and Zhao Xiaoxi, "Alpha-Clustering and Alpha-Decay", *Proc. of Int. Nucl.Phys.Conf., Harrogate* , U.K.,1986

38. G.Dodig-Crnković, F.A.Janouch, R.J.Liotta and Zhao Xiaoxi, "The Continuum and the Alpha-particle Formation", *Physica Scripta*, **37**(1988)523

39. G.Dodig-Crnković, F.A.Janouch and R.J.Liotta, "Alpha–particle Formation in Heavy Nuclei", To be published in *Conf.Proc.Int.Symp.on Modern Developments in Nuclear Physics* , Novosibirsk, 1987 World Publ., Singapore

40. G.G.Dussel, R.J.Liotta and R.P.J.Perazzo, "Clusters of Nucleons as Elementary Modes of Excitation in Nuclei", *Nucl.Phys.* **A388**(1982)606

41. A.R.Edmonds, *Angular Momentum in Quantum Mechanics*, Princeton University Press, 1957

42. J.Eichler, Quadruples, "Four Nucleon Correlations and Alpha Rotator Structure", *Proc.of the Symp. held at the Phys. Division, University of Marburg* , 1972.

43. J.Eichler, M.Yamamura, "Quadrupling and Pairing in the Shell Model", *Nucl.Phys.* **A182**(1972)33

44. J.P.Eliott and D.A.Lea, "Pairing Force with Neutrons and Protons", *Phys.Lett.* **19**(1965)291

45. L.Ferreira, R.Liotta, C.H.Dasso, R.A.Broglia and A.Winther, "Spatial Correlations of Pairing Collective States", *Nucl.Phys.* **A426**(1984)276

46. T.Fliessbach and S.Okabe, "Surface Alpha-Clustering in the Lead Region", *Z.Phys.* **A320**(1985)289

47. B.H.Flowers and M.Vujičić, "Charge-Independent Pairing Correlations", *Nucl.Phys.* **49**(1963)586

48. T.Fliessbach and H.J.Mang, "On Absolute Values of Alpha–Decay Rates", *Nucl.Phys.***A263** (1976)75

49. T.Fliessbach, H.J.Mang and J.O.Rasmussen, "Normalized Shell Model Alpha–Decay Theory Applied to Unfavoured Decay", *Phys.Rev.* **C13**(1976)1318

50. T.Fliessbach, "The Effective Single Particle Operator for Reactions with Alpha-Clusters", *Z.Physik.* **A294**(1980)79

51. T.Fliessbach, "The Energy and Target Dependence of the Real Alpha-Nucleus Potential", *Nucl.Phys.* **A315** (1979)109

52. T.Fliessbach, "A Model Calculation of the Asymptotic Reduced Alpha-Amplitude", *Nucl.Phys.* **A285** (1977)262

53. Y.K.Gambhir, P.Ring and P.Schuck, "Nuclei: A Superfluid Condensate of Alpha-Particles? A Study within the Interacting-Boson Model", *Phys.Rev.Lett.* **51** (1983)1235

54. B.Giraud, "Generator Coordinate, Alpha-Cluster, Solution of the Hill-Wheeler Equation...usw.", *Four Nucleon Correlations and Alpha Rotator Structure* .Proc.of the Symp.held at the Phys.Div., University of Marburg, 1972.

55. W.K.Glendening, "Four-Particle Matrix Elements", *Phys.Rev.* **127**(1962)923

56. N.K.Glendening and Kichinosuke Harada, "Shell-Model Calculation for 212Po", *Nucl.Phys.* **72** (1965)481

57. M.L.Goldberger and K.M.Watson, "The Lifetime and Decay of Virtual States", *Collision Theory* , Wiley, N.Y., 1964., p.424

58. I.S.Gradshteyn and I.M.Ryzhik, *Table of Integrals, Series, and Products* , Academic Press, 1965

59. B.Gyarmati, K.F.Pal and T.Vertse, "On the Shape of the Alpha-potential in Direct Alpha Transfer", *Phys.Lett.* **104B**(1981)177

60. G.C.Hanna, "Alpha-Radioactivity", *Experimental Nuclear Physics*, Vol.III, Ed.E.Segre, Wiley, N.Y., 1959

61. O.Hansen, "Spectroscopic Information from Few Particle Transfer Reactions", *Proc. Int.Conf. on Properties of Nucl. States, Montreal* , 1969., p.421

62. K.Harada, "Alpha-Particle Reduced Widths in Heavy Nuclei", *Progr.Theor.Phys.* **26**(1961)667

63. K.Harada and Elisabeth.A.Raucher, "Unified Theory of Alpha Decay", *Phys.Rev.* **169**(1968)818

64. H.Hasan and C.S.Warke, "Semiclassical Analysis of Alpha–Transfer in 212Po", *Clustering Aspects of Nuclear Structure and Nuclear Reactions*, Winnipeg, 1978.

65. G.H.Herling and T.T.S.Kuo, "Two-Particle States in 210Pb, 210Bi and 210Po with Realistic Forces", *Nucl.Phys.* **A181**(1972)113

66. M.W.Herzog, R.J.Liotta, L.J.Sibanda, "Pair Clustering and Giant Pairing Resonances", *Phys.Rev.***C31** (1985)259

67. S.Holan, "On Surface Clustering and Pauli Effects in Alpha Decay", *Preprint JINR-E4-83-337*

68. P.E.Hodgson, "Alpha Clustering in Lead Nuclei", *Growth Points in Nuclear Physics* , p.63, Pergamon Press, 1981

69. J.J.Hogan, "Alpha Particle Clusters in Complex Nuclei: A Semi-empirical Calculation", *Clustering Aspects of Nuclear Structure and Nuclear Reactions* (Winnipeg, 1978)

70. F.Iachello, "Unification of Nuclear Structure Models", *Nucl.Phys.* **A421**(1984)97c

71. F.Iachello, "Algebraic Methods for Molecular Rotation-Vibration Spectra", *Chem.Phys.Lett.* **78**(1981)581

72. F.Iachello and A.D.Jackson, "A Phenomenological Approach to Alpha-Clustering in Heavy Nuclei", *Phys.Lett.* **108 B**(1982)151

73. G.Igo, "Optical-Model Analysis of Excitation Function Data and Theoretical Reaction Cross Sections for Alpha Particles", *Phys.Rev.* **115**(1959)1665 74. Daphne F.Jackson, M.Rhoades-Brown, "Theories of Alpha-Decay", *Ann. Phys.* **105**(1977)151

75. Daphne F.Jackson, M.Rhoades-Brown, "The Alpha-Transfer to Alpha-Decaying Nuclei", *Nucl.Phys.* **A286**(1977) 354

76. Daphne F.Jackson, M.Rhoades-Brown, "Antisymmetrization in Alpha Decay and Alpha Transfer", *J.Phys.G* **4** (1978)1441

77. Daphne F.Jackson, M.Rhoades-Brown, "Alpha-Particle Scattering Near the Coulomb Barrier", *Nucl.Phys.* **A226**(1976)61

78. Daphne F.Jackson, M.Rhoades-Brown, "A Consistent Description of the Nuclear Alpha-Decay Process", *Nature* **267**(1977)593

79. Daphne F.Jackson, "Interpretation of Alpha Clustering in Nuclei", *J.Phys.G* **4** (1978)1287

80. A.Jaffrin, "Quartet States in the sd and fp Shells", *Nucl.Phys.* **A196**(1972)577

80.1 R.D.Lawson, *Theory of the Nuclear Shell Model*, Oxford University Press, 1980, 203

80.2 R.D.Lawson, M.Goeppert-Mayer, "Harmonic Oscillator Wave Functions in Nuclear Spectroscopy",*Phys.Rev.* **117** (1960)174

81. F.A.Janouch and R.J.Liotta, "Nuclear Field Theory Approach to Alpha-Decay", *Phys.Lett.* **82B**(1972)329

82. F.A.Janouch and R.J.Liotta, "Nuclear Structure and Relative Alpha-Decay Width of 212At within the Framework of the Nuclear Field Theory", *Nucl.Phys.* **A334**(1980) 427

83. F.A.Janouch and R.J.Liotta, "Role of Configuration Mixing on Absolute Alpha-Decay Width in Po Isotopes", *Phys.Rev.* **C25**(1982)2123

84. F.A.Janouch and R.J.Liotta, "Influence of Alpha-Cluster Formation on Alpha-Decay", *Phys.Rev.* **C27** (1983)896

85. R.D.Lawson, *Theory of the Nuclear Shell Model* , Oxford University Press, 1980

86. R.J.Liotta and C.Pomar, *Nucl.Phys.* **A382**(1982)1

87. H.J.Mang, "Alpha Decay", *Ann.Reviews Nucl.Sci.* **14**(1964)1

88. H.J.Mang, "Calculation of Alpha-Transition Probabilities", *Phys.Rev.***119**(1960)1069

89. T.Marumori and K.Suzuki, "Four-Body Correlations in Light Nuclei", *Nucl.Phys.* **A106**(1968)610

90. L.Marques, "Identity between the Widths from Alpha Decay and Alpha Elastic Scattering", *Phys.Rev.* **C25** (1982)1065

90.1 M.Moshinsky, " Transformation Brackets for Harmonic Oscillator Functions", *Nucl.Phys.***13**(1959)104

91. C.P.Panos, "The Non-Unitary Model Operator Approach with Realistic Potentials and its Applications to ^4He", *J.Phys.G.Nucl.Phys.***12**(1986)495

92. A.A.Pilt, "Cluster States in Light Nuclei", *Clustering Aspects of Nuclear Structure and Nuclear Reactions* (Winnipeg, 1978)

93. W.T.Pinkston, "Clustering of Nucleons in the Nuclear Surface", *Phys.Rev.***C29**(1984)1123

94. H.M.A.Radi, A.A.Shibab-Eldin and J.O.Rasmussen, "Configuration-mixed-shell-model relative Alpha-decay Rate", *Phys.Rev.* **C15**(1977)1917

95. J.O.Rasmussen, "Simplified Shell-Model Calculation of Alpha Decay Rate near ^{208}Pb, *Nucl.Phys.* **44**(1963) 93

96. J.Raynal, M.A.Melkanoff and T.Sawada, "Nuclear-Spectroscopy Calculations including Continuum States and Application to the Giant Resonance of ^{16}O", *Nucl.Phys.* **A101**(1967)369

97. D.Robson, "Alpha Particle Clustering and the Quark Structure of Elementary Particles", *Clustering Aspects of Nuclear Structure and Nuclear Reactions* (Winnipeg, 1978)

98. A.Sandulescu, "Alpha Reduced Widths on Superfluid Model", *Nucl.Phys.***48**(1963)345

99. A.Sandulescu, I.Silisteanu and R.Wünch, "Alpha Decay within Feshbach Theory of Nuclear Reactions", *Nucl.Phys.***A305**(1978)205

100. S.Sasaki, S.Suekane and I.Tonozuka, "Alpha-Particle Type Four-Body Correlation in 212Po", *Nucl.Phys.* **A147**(1970)45

101. Hiroshi Sato, "Systematics of the Alpha Q-values", *Clustering Aspects of Nuclear Structure and Nuclear Reactions* (Winnipeg, 1978)

102. K.W.Schmid, "The Multi Configuration Hartree Fock Model and Intrashell Quartet States in sd-shell Nuclei", *Four Nucleon Correlations and Alpha Rotator Structure* , Proc.of the Symp. held at the Phys.Division, University of Marburg, 1972.

103. A.de-Shalit and I.Talmi, *Nuclear Shell Theory* , Academic Press, N.Y. and London, 1963

104. M.R.Schmorak, "Systematics of Nuclear Level Properties in the Lead Region", *Nuclear Data Sheets* **31** (1980)283

105. B.Silvestre-Brac, "The Cluster Model and the Generalized Brody-Moshinsky Coefficients", *J.Physique* **46** (1985)1087

106. V.G.Soloviev, "Teorija Alfa-raspada", *Teorija Atomnogo Yadra. Yadernie Modeli* .p.215, Energoizdat, Moskva, 1981

107. V.G.Soloviev, "Effect of Pairing Correlations on the Alpha Decay Rates", *Phys.Lett.***1**(1962)202

108. V.G.Soloviev, "Vlijanie parnyh korelacij na vjerojatnosti alfa-raspadov", *Teorija sloznyh yader* , p.242

109. T.Steinmayer, W.Sünkel and K.Wildermuth, "The Ground- state Alpha-decay of 212Po", *Phys.Lett.* **125B** (1983)437

110. S.Szpikowski and M.Trajdos, "Four-particle Correlations in Nuclei on the $sd-f_{7/2}$ Levels", *Acta Phys.Polonica* **B15**(1984)673

111. S.Szpikowski and M.Trajdos, *Nucl.Phys.* **A272** (1976)155

112. S.Szpikowski, "Four-Body Microscopic Interpretation of the Interacting Boson Model", *Proc. of Int. Spring Seminar on Nucl. Phys.: Microscopic Approaches to Nuclear Structure Calculations* , Ed. A.Covello, Sorento, 1986

113. I.Tonozuka and A.Arima, "Surface Alpha-Clustering and Alpha-Decays of 212Po", *Nucl.Phys.* **A323**(1979)45

114. R.M.De Vries, "Alpha Decay and Alpha Transfer Reactions", *Nukleonika* **24**(1979)67

115. R.M.De Vries, D.Shapira, N.R.Clover, H.E.Gove, J.D.Garrett and G.Sorensen, "A Search for Alpha-Particle Vibrational States in 208Pb", *Phys.Lett.* **67B**(1977)19

116. R.M.De Vries, D.Shapira, W.G.Davies, G.C.Ball, J.S.Forster and W.Mc Latchie, "Inverse Alpha-Decay via the Reaction $^{208}Pb(^{16}O, ^{12}C)^{212}Po$", *Phys.Rev.Lett.***35**(1975)836

117. R.M.De Vries, J.S.Lilley and M.A.Franey, "Absolute Reduced Alpha-Widths in the Lead Region", *Phys.Rev.Lett.* **23**(1976)481

118. A.H.Wapstra, "The 1977 Atomic Mass Evaluation", *Atomic Data and Nuclear Data Tables* **19** (1977)175

119. S.Weinberg, *The First Three Minutes, A modern view of the origin of the universe* , Fontana Paperbacks, 1986

120. H.H.Wolter, "Proton-Neutron Correlation in the Framework of HFB Theory", *Proc. of the 15th Int. Meeting of Nucl. Physicists on Nuclear Structure, Herceg Novi,* 1970 Ed. A.H.Kukoc

121. D.Zawischa, "On the Origin of Odd-even-staggering of Nuclear Charge Radii", *Phys.Lett.* **155B** (1985)309

122. D.Zawischa, U.Regge and R.Stapel, "Four-body Correlations, Four-body interaction and the Staggering of Nuclear Charge Radii", *Proc.of Int.Spring Seminar on Nucl.Phys. Microscopic Approaches to Nuclear Structure Calculations* , Ed. A.Covello, Sorento, 1986

123. A.P.Zuker, "Four Particle Correlations and the Shell-model Four Nucleon Correlations and Alpha Rotator Structure", *Proc.of the Symp. held at the Phys.Division, University of Marburg* , 1972.

SUMMARY

The microscopic derivation of the four-particle cluster formation amplitude as well as absolute alpha-decay width within the extended shell model is presented which enables us to study clustering features and the alpha-decay process. The half-life for the α-decay of ^{212}Po is calculated within a harmonic oscillator representation. Clustering induced by the nuclear interaction appear by considering a large configuration space. The role of clusterisation in the alpha-decay process is clearly demonstrated. It was shown that as soon as four-particle cluster is formed on the surface of the nucleus a broad region appears in which the ratio between theoretical half-lives and experimental ones is independent on the channel radius. That is the important result itself which indicates that the old problem of strong dependence of alpha-decay widths on arbitrary parameter of channel radius can be solved by describing the alpha-decay as the disintegration of the initial state in which alpha clustering occurs. The role of proton-neutron interaction proved to be decisive in the formation of the stable four-particle cluster.

SAŽETAK

Načinjen je mikroskopski izvod amplitude formiranja 4-čestičnog klastera kao i apsolutne širine alfa-raspada u okviru proširenog ljuskastog modela, koji omogućuje proučavanje svojstava klasterizacije a zatim i α-raspada. Vrijeme poluživota za α-raspad ^{212}Po izračunato je u reprezentaciji harmoničkog oscilatora. Klasterizacija uzrokovana nuklearnom interakcijom pojavljuje se uzimanjem u obzir golemog konfiguracijskog prostora. Uloga klasterizacije u procesu alfa-raspada je jasno ocrtana. Pokazuje se da formiranje 4-čestičnog klastera na površini jezgre ima za posljedicu neovisnost omjera teorijskog i eksperimentalnog vremena poluživota o radijusu kanala. To je po sebi važan resultat koji pokazuje da se stari problem jake ovisnosti α-širina o proizvoljnom parametru radiusa kanala može riješiti ako α-raspad shvatimo kao proces dezintegracije početnog klasterskog stanja. Pokazano je da za formiranje stabilnog klastera proton-neutron interakcija igra odlučujuću ulogu.

Curriculum Vitae

1955. born in Doboj, Yugoslavia.

1974. finished secondary school in Kutina, Yugoslavia.

1979. recieved the B.S. in physics from Faculty of Natural Sciences, University of Zagreb, Yugoslavia.

1979. joined the Theoretical Nuclear Physics Group at Rudjer Bošković Institute, Zagreb.

1983. recieved the M.S. degree in theoretical nuclear physics from University of Zagreb, Yugoslavia.

1984. two-month stays at Niels Bohr Institute (NBI)- Copenhagen and AFI- Stockholm. Alpha-decay and alpha-clustering project.

1985-1988. work on Ph.D. thesis at AFI-Stockholm with shorter stays at NBI-Copenhagen.

List of Publications

1. Gordana Dodig, S.Brant, V.Paar, B.K.Koene, R.Chrien, "Systematics of N=79 Nuclei", *Fizika* **11**, Supplement 1 (1979)9

2. V.Paar, S.Brant, Gordana Dodig, Lj.Udovčić, G.Leander, "The structure of Nilsson States and Rotational Bands Generated by the Spherical Particle–vibration Coupling in Odd–A Nuclei",*Fizika* **11**, Supplement 1 (1979)9

3. V.Paar, S.Brant, Gordana Dodig, Lj.Udovčić, G.Leander, "The structure of Nilsson States and Rotational Bands Generated by the Spherical Particle–vibration Coupling in Odd–A Nuclei", *Verhandlungen der Deutschen Physikalischen Gessellshaft* (1979)683

4. G.Dodig-Crnković, F.A.Janouch and R.J.Liotta, "Alpha–Clustering and Absolute Alpha–Decay Widths in Spherical Nuclei", *Phys.Lett.***139B**(1984)143

5. G.Dodig-Crnković, F.A.Janouch and R.J.Liotta, "Absolute Alpha–Decay Widths in Po Isotopes", *Proc.Int.Symp. on In-beam Spectroscopy, Debrecen* , 1984

6. G.Dodig-Crnković, F.A.Janouch, R.J.Liotta and L.J.Sibanda, "Absolute Alpha-Decay Rates in Po Isotopes", *Nucl.Phys.* **A444**(1985)419

7. G.Dodig-Crnković, F.A.Janouch and R.J.Liotta, "Absolute Alpha-Decay Widths", *Proc. Yugoslav Meeting of Nuclear and Particle Physicists, Portoroz* , 1985

8. G.Dodig-Crnković, F.A.Janouch, R.J.Liotta and Zhao Xiaoxi, "Alpha-Clustering and Alpha-Decay", *Proc. of Int. Nucl.Phys.Conf., Harrogate* , U.K.,1986

9. G.Dodig-Crnković, F.A.Janouch, R.J.Liotta and Zhao Xiaoxi, "The Continuum and the Alpha-particle Formation", *Physica Scripta* ,**37**(1988)523

10. G.Dodig-Crnković, F.A.Janouch and R.J.Liotta, "Alpha–particle Formation in Heavy Nuclei", To be published in *Conf.Proc.Int.Symp.on Modern Developments in Nuclear Physics* , Novosibirsk, 1987 World Publ., Singapore

EPILOGUE

This is not the end.
This is not even the beginning of the end.
This is, perhaps, the end of the beginning.

W.Churchill