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The Continuum and the Alpha-particle Formation

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Abstract

The absolute α-decay width of ^{212}Po is calculated within a harmonic oscillator representation. Clustering features induced by the nuclear interaction appear by considering a large configuration space. The role of the neutron–proton interaction is analysed and a reasonable account of the experimental alpha-decay width is given.

The systematics of alpha-transfer and alpha-decay processes throughout the periodic table points to the existence of four-nucleon dynamical clusters on the nuclear surface. However, microscopic studies of alpha-clustering and alpha-decay are difficult because one has to consider simultaneously the movement of the nucleons which eventually constitute the alpha particle both inside and outside the nucleus. This means that one needs to describe at the same time the behaviour of the nucleons both in the bound and in the unbound states (continuum). The dynamics of these four bodies moving in the nuclear field is quite a complicated problem. To simplify it one tempting possibility is to use only the main single particle levels to describe the wave functions of the neutrons and the protons [1]. By doing this the calculated absolute alpha-decay widths are usually wrong by several orders of magnitude when comparing with experimental data. The physical reason for this enormous discrepancy is that the description of the clustering of the alpha-particle beyond the nuclear surface requires the inclusion of the continuum part of the single-particle representation. In other words, one has to include many configurations to give a reasonable account of the alpha-cluster. Indeed, one knows that the absolute alpha-decay width increases strongly with the number of configurations [2, 4, 9].

Recent calculations show that by including the neutron–neutron (nn) and proton–proton (pp) interactions within a large single-particle space, the calculated absolute alpha-decay width is still away from the corresponding experimental value by almost two orders of magnitude. The inclusion of the neutron–proton (np) interaction reduces this discrepancy to about one order of magnitude. All these calculations were done using Wood–Saxon radial single-particle wave functions [3, 4]. Since no analytical integration is possible in this case, the number of configurations included was not very big due to computational limitations. However, one knows that in other fields where the continuum is also important (e.g., giant resonances) a harmonic oscillator representation is adequate enough [10]. With this in mind we will try to analyze the absolute alpha-decay width in spherical nuclei using a harmonic oscillator representation. In this way we will be able to enlarge the dimension of the shell-model space clarifying the question of whether the missing orders of magnitude mentioned above is due to the insufficient number of configurations included in the calculation or to other effects. We will study the deay of ^{212}Po which has two protons and two neutrons outside the ^{208}Pb core. As in Ref. [3], we will include the nn, pp and np interactions through the corresponding correlated (pairing) states. One can then write

$$|^{212}\mathrm{Po(gs)}\rangle = A|^{210}\mathrm{Pb(gs)} \otimes {}^{210}\mathrm{Po(gs)}\rangle + B|^{210}\mathrm{Bi}(0_1^+) \otimes {}^{210}\mathrm{Bi}(0_1^+)\rangle \quad (1)$$

where A and B are normalization constants and the two particle wave functions corresponding to ^{210}Pb(gs), ^{210}Po(gs) and ^{210}Bi(0_1^+) can be written as

$$\Psi(r_1 r_2; n\lambda) = \sum_{k_1 k_2} X(k_1 k_2; n\lambda)\, \mathscr{A}\,(\phi_{k_1}(r_1)\phi_{k_2}(r_2))_\lambda . \quad (2)$$

In our calculation the wave function amplitudes X in eq. (2) were obtained by diagonalizing the TDA equation with a surface delta interaction (SDI). The strength of the interaction was adjusted so that one obtains the experimental ground state values, i.e., ^{210}Pb(gs) for the nn and ^{210}Po(gs) for the pp case, or the state ^{210}Bi(0_1^+) for the np interaction. This state has not been observed experimentally yet, but we assumed that it lies at 5 MeV of excitation energy above the ^{210}Bi(gs). Details of these calculations are given in Ref. [3]. The basis states used to describe the mother nucleus ^{212}Po(gs) in eq. (1) are, in principle, not independent of each other. But one can calculate the overlap between them and therefore also the amplitudes A and B. Using the multistep shell-model method [5] we found that most of the contribution to the overlap is given by the single-particle states in the major shells close to the Fermi level in eq. (2). By increasing the space and including more shells, the alpha-decay width increases as we will show below, but the overlap remains practically constant and equal to 0.11. Also the calculated energy of the state $|^{212}\mathrm{Po(gs)}\rangle$ is not sensitive to the inclusion of high lying shells. This energy agrees with the corresponding experimental value within 100 keV while we obtained $A = 0.96$, $B = 0.30$ (note that $A^2 + B^2 = 1.01$, which reflects the small overlap between the basis states).

Although the values of A and B are rather insensitive to

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the inclusion of the high lying shells, one has to consider that the wave function (1) describes the bound properties of the mother nucleus. However, in the decay process the unbound states may become important. In fact, one can take the extreme point of view that the unbound states are determinant in alpha decay and consider this decay as a strongly asymmetric fission process [6]. In our case, we may equilibrate this unbalance between the description of the bound and unbound properties by considering B as an effective parameter and allowing for a small variation around the "bound" value $B = 0.30$.

Once the wave function (1) was obtained, we calculated the alpha decay width by assuming that the alpha decay proceeds through the classical two-step mechanism proposed by Gamow. Namely, we assume that the four nucleons that constitute the alpha-particle are first clustered in a point R on the nuclear surface and then the alpha particle thus formed penetrates the Coulomb barrier. The corresponding alpha-decay width is

$$\Gamma_L(R) = 2\gamma_L^2 P_L(R) \tag{3}$$

where $P_L(R)$ is the penetration factor through the Coulomb barrier from the point R and

$$\gamma_L = \left(\frac{\hbar^2 R}{2\mu}\right)^{1/2} F_L \tag{4a}$$

where

$$F_L(R) = \int (\varphi_d \varphi_\alpha Y_L)^* \varphi_m d\tau \tag{4b}$$

is the alpha-particle formation amplitude at the point R. In eq. (4) we use a standard notation [3], $\varphi_d(\varphi_m)$ is the wave function of the daughter (mother) nucleus and φ_α is the intrinsic wave function of the alpha-particle. That is

$$\varphi_{J_\alpha=0} = \left(\frac{2\beta^{3/2}}{(1/2)!}\right)^{3/2} \exp\left[-\frac{\beta}{2}(\xi_1^2 + \xi_2^2 + \xi_3^2)\right] \times (4\pi)^{-3/2} \chi(12)\chi(34) \tag{5}$$

where

$$\xi_1 = \frac{1}{\sqrt{2}}(r_1 - r_2),$$

$$\xi_2 = \frac{1}{\sqrt{2}}(r_3 - r_4),$$

$$\xi_3 = \tfrac{1}{2}(r_1 + r_2 - r_3 - r_4)$$

and χ is the spin wave functions while $\beta = 0.574\,\mathrm{fm}^{-2}$ [9].

It was further assumed that the particles 1 and 2 (3 and 4) are neutrons (protons). In the case of alpha-decay only the singlet component of the spin function contributes. In fact, it is only for the singlet component of the two-particle wave function (2) that the nucleons positioned in r_1 and r_2 get clustered when the configuration space is enlarged [7]. This is indeed the connecting point between clustering and alpha-decay. By increasing the shell-model space, the two particles moving in the singlet state get clustered in the wave function φ_m and this clustering, in turn, increases the overlap function $F(R)$ as seen in eq. (4). Using the harmonic oscillator representation the integral (4) can be performed analytically, thus making it possible to include very high lying states within a reasonable CPU time in our VAX 11-7800 computer.

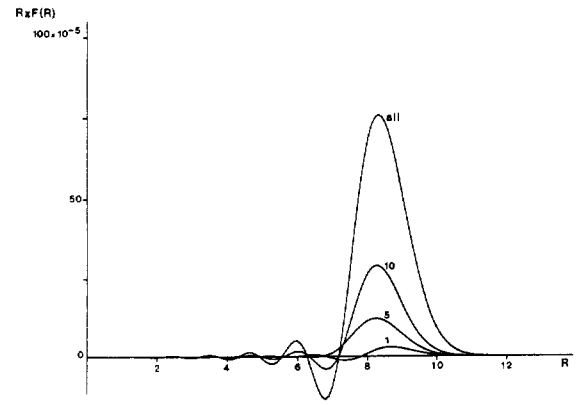

Fig. 1. Formation amplitude $F(R)$ times R (in $\mathrm{fm}^{-1/2}$) as a function of the distance to the center of the mother nucleus R (in fm) calculated for the leading proton–proton configuration and for different numbers (on top of each curve) of neutron–neutron configurations.

The clustering features of $|^{210}\mathrm{Pb(gs)}\rangle$ as well as $|^{210}\mathrm{Po(gs)}\rangle$ and $|^{210}\mathrm{Bi}(0_1^+)\rangle$ are similar to those shown in Refs. [7, 8]. As in these references, the clustering appears only when the number of configurations is increased.

The clustering of the alpha-particle as a function of the number of configurations can better be seen by analyzing the formation amplitude (4) directly.

In Fig. 1 we show the amplitude $F(R)$ times R as a function of R using only one proton single-particle state (the state $0h_{9/2}$, which is the closest to the Fermi level) and neutron single-particle states increasing from one state up to all states within $\Delta N = 4$ excitations in our harmonic oscillator representation.

Similarly, in Fig. 2, we included only the leading neutron configuration (the state $1g_{9/2}$) while the number of proton configurations was increased.

In both figures one sees the same effect, namely that the formation amplitude strongly increases in the surface of the mother nucleus and even outside it. In these figures the values of A and B are those obtained from the calculation of the ground state energy of ^{212}Po. As mentioned above, a way to see the importance of the neutron-proton interaction in the formation of the alpha-particle is to increase the value of B maintaining the normalization condition $A^2 + B^2 = 1$. The importance of both the neutron-proton interaction and the configuration mixing is seen in Fig. 3. Without that interaction (i.e. $B = 0$) the form factor $F(R)$ is small even with all configuration included. Even more important, already with

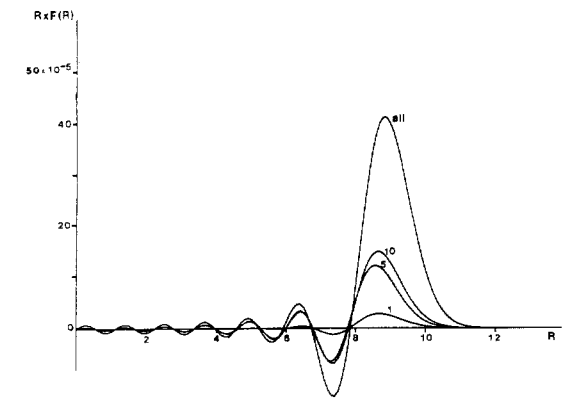

Fig. 2. The same as Fig. 1 for the leading neutron–neutron configuration and changing the number of proton–proton configurations.

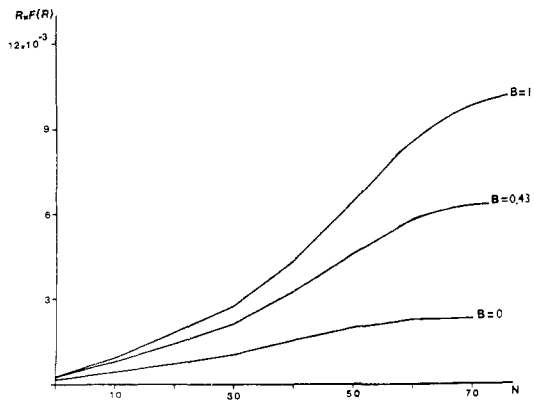

Fig. 3. Dependence of $R \times F(R)$ (in $\mathrm{fm}^{-1/2}$) on the number of configurations N for $R = 9$ fm and different values of the parameter B.

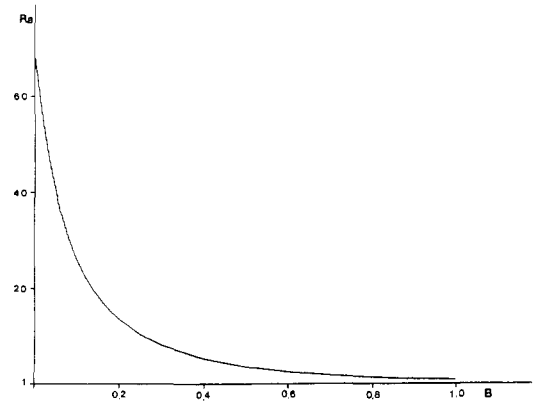

Fig. 5. Ratio Ra = $T_{1/2}$ (theory)/$T_{1/2}$ (exp) as a function of B for $R = 9$ fm.

50 configurations one reaches the "saturation" of the calculated value of $F(R)$. Perhaps surprisingly, at the other end, with only neutron-proton interaction present ($A = 0$, $B = 1$) the form factor does not reach a maximal value even within the $\Delta N = 4$ space used in the calculations. One may argue that the form factor calculated with a SDI diverges as the number of configuration is increased. This would indeed be the case if the value of the SDI strength would be the same independently of the number of configurations. However, in all the cases presented in this paper we adjusted the strength so as to fit the energies of the lowest 0^+ states, as mentioned above, for each subspace considered. This is the reason why for the case $A = 1$, $B = 0$ the form factor converges to a final value already within 50 configurations. For the "'physical" case $A = 0.90$, $B = 0.43$ the form factor converges to its final value just within all the configurations included in our $\Delta N = 4$ space.

We can now analyze the alpha-decay half life and compare it with the experimental value. The Gamow interpretation of alpha-decay, which leads to eq. (3), requires that there exists a region of space around the nuclear surface where the width is independent of the distance R. However, both the penetration factor $P_L(R)$ and the reduced alpha-decay width $\gamma_L(R)$ depends very strongly on R. But by including all the configurations we found that the width $\Gamma(R)$ is almost independent on R within a large interval around the nuclear surface and even beyond it, as seen in Fig. 4. A striking feature of this figure is that the calculated half live (i.e., the width $\Gamma(R)$) becomes less dependent upon R on the nuclear surface as B is increased. This is a natural consequence of the fact that the neutron–proton interaction increases the alpha-clustering. In other words, the more the clustering features are pronounced in the initial state the better are the Gamow assumptions fullfilled. This is in itself an important result, supporting the original Gamow theory of alpha-decay. Yet, the calculated half-life is still too small by about one order of magnitude. Only by increasing the neutron–proton interaction towards rather unrealistic values one approaches the experimental half-life, as seen in Fig. 5.

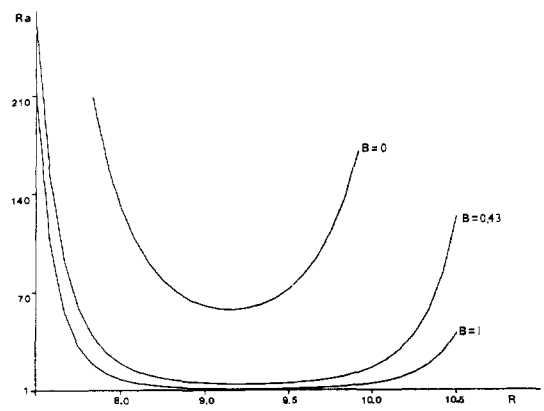

Fig. 4. Ratio Ra = $T_{1/2}$ (theory)/$T_{1/2}$ (exp) as a function of R for different values of B.

In conclusion we have shown in this paper that the inclusion of a very large configuration space to describe the clustering of the alpha particle in alpha-decay is necessary. The calculated decay width increases by more than four orders of magnitude from the case when only one (main) configuration is included to the case where the number of configurations is such that the calculation converges to the final value. Yet, the calculated width is too small by about a factor of five when the wave-function corresponding to the bound state of the mother nucleus is used. Only by increasing the neutron–proton interaction our calculated width approaches the corresponding experimental value. This suggests that our treatment of the neutron–proton interaction may be too schematic. A more detailed analysis of this interaction would perhaps be required to adjust this relatively small difference.

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