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AN EXACT SHELL-MODEL TREATMENT OF α-CLUSTERING AND ABSOLUTE α-DECAY

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Abstract: The alpha clustering and corresponding alpha decay of ^{212}Po is studied within the framework of a multistep shell-model method. All interactions among the four nucleons that constitute the alpha particle are included and a large single-particle representation is used. It is found that this approach is not sufficient to explain the available experimental data.

1. Introduction

There is a renewed interest in α-decay due to the "exotic" decay modes recently discovered [1]), as well as the connected question of clustering of nucleons in nuclei [2-5]). The study of the formation of the α-particle on the nuclear surface and its penetration through the Coulomb barrier have been analyzed from different points of view. In particular, it was argued that the Pauli principle acting among the nucleons in the core and those in the α-cluster is so important that one can not speak of "penetration" of the α-cluster as a whole [6]). But if one is able to describe the formation of the α-particle well outside the daughter nucleus one can neglect the Pauli principle [7]) and apply the old two-step mechanism proposed by Gamow at the very early years of quantum mechanics. This approach has been successful, not only in explaining α-decay but also, rather unexpectedly, the recently discovered emission of heavy clusters [8]).

In the original Gamow approach it was assumed that the α-particle is already formed at a given point on the nuclear surface and the problem of clustering is disregarded. The great success of this approach was the introduction of the quantum mechanical concept of penetration through a potential barrier. But only later the concept of "formation probability" was studied in detail and properly defined [9]). Together with the simultaneous appearance of the nuclear shell model, this allowed a complete microscopic approach to take place. Thus it was soon clear that the nuclear short-range interaction (pairing), acting through many shell-model configurations, highly enhances the calculated α-decay width [10]). It was also found that the physical feature behind this enhancement was that, through the configuration mixing,

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the pairing interaction clusters the two neutrons and the two protons on the nuclear surface [11,12]). In these calculations large single-particle representations were used and through them the calculated decay widths were enhanced by several orders of magnitude. Even so, the corresponding experimental values were still larger by about one order of magnitude. One possibility for this discrepancy was that only the neutron–neutron and proton–proton interactions were included in the calculations. The importance of the neutron–proton interaction was recently studied in a rather schematic way [2,5]). The novel feature of this approach was the introduction of a high-lying pair collective mode, i.e. the giant pairing resonance [13]) which is now under experimental investigation [14]). The understanding of the relation between α-decay and α-clustering was considerably improved by using this rather schematic approach. But a proper treatment of the neutron–proton interaction in heavy nuclei requires an effort beyond the calculations of refs. [2,5]). One may expect that within such a treatment the agreement between theory and experiment would be improved. With this in mind it is the purpose of this paper to treat all two-body interactions among neutrons and protons involved in α-decay exactly within the framework of the shell model.

The formalism is described in sect. 2, the application in sect. 3, and a summary and conclusions are given in sect. 4.

2. Formalism

The analysis of a system consisting of two neutrons and two protons moving outside a closed-shell core can conveniently be made within the framework of the shell model. Once the interaction among the particles has been chosen the corresponding shell-model equations are not difficult to solve. However, the shell-model dimensions are usually very large because the interaction connects even those components which may be very far (in energy) from each other. As a result the diagonalization of the shell-modell matrix becomes a difficult and rather meaningless task.

An alternative to the shell-model procedure is to use as basis elements correlated states. Since a large part of the correlations are then included already in the basis one may avoid the spreading of the wave function into many small components and drastic truncations of the correlated basis can be performed. But the use of correlated bases to describe nuclear systems may, in general, present the problem that the Pauli principle is not properly taken into account and, besides, one may count more than once the same states in the basis. There are methods to deal with this problem. Among these, the multistep shell-model method (MSM) [15]) has the advantage that one describes the system in terms of previously calculated systems. In our case the most general MSM basis would be that in which the basis elements are expressed in terms of two-proton (α_2), two-neutron (β_2) and neutron–proton (γ_2) correlated states. The α-particle system, that is the core plus an α-particle, can

then be written as

$$|\alpha_4\rangle = \sum_{\alpha_2\beta_2} X(\alpha_2\beta_2;\, \alpha_4) P^+(\alpha_2) P^+(\beta_2)|0\rangle + \sum_{\gamma_2\gamma_2'} X(\gamma_2\gamma_2';\, \alpha_4) P^+(\gamma_2) P^+(\gamma_2')|0\rangle\,, \quad (1)$$

where the operators $P^+(\alpha_2)$ create two-particle states, i.e.

$$P^+(\alpha_2) = \sum_{ik} X(ik;\, \alpha_2) c_i^+ c_k^+\,, \quad (2)$$

c^+ is the single-particle creation operator and X is the two-particle wave function amplitude. Although we use the same symbol for the amplitudes in eqs. (1) and (2), no confusion can arise since the arguments in the quantity X clearly indicate which case is being considered.

The amplitudes X in eq. (1) are generally not well defined quantities, because the basis elements may not be orthogonal to each other and, moreover, they may span a space with a dimension which is smaller than the number of basis states. That is, the MSM basis may be non-orthogonal and overcomplete. To solve this problem within the MSM, one calculates the overlap among all the basis states (the so-called metric matrix). Usually the metric matrix has a form which is very similar to the dynamical matrix.

The general expression given by eq. (1) may be useful in cases where one knows that some γ_2-states play an important role. Their importance would then be manifested in that the corresponding MSM wave functions will have large components in those states. However, in the case that we want to describe it would be more natural to restrict the basis elements to states of type α_2 and β_2 only. In this way the Pauli principle is not effective and the α-particle system can be written as

$$|\alpha_4\rangle = \sum_{\alpha_2\beta_2} X(\alpha_2\beta_2;\, \alpha_4) P^+(\alpha_2) P^+(\beta_2)|0\rangle\,. \quad (3)$$

Since the basis elements in eq. (3) are orthonormal to each other, the amplitudes X can directly be evaluated from the dynamical matrix which in this case reads

$$W(\alpha_4) X(\alpha_2\beta_2;\, \alpha_4) = (\omega(\alpha_2) + \omega(\beta_2)) X(\alpha_2\beta_2;\, \alpha_4 + \sum_{\alpha_2'\beta_2'} M(\alpha_2\beta_2;\, \alpha_2'\beta_2';\, \alpha_4) X(\alpha_2'\beta_2';\, \alpha_4)\,, \quad (4a)$$

where, with standard notation,

$$M(\alpha_2\beta_2;\, \alpha_2'\beta_2';\, \alpha_4) = \hat{\alpha}_2\hat{\beta}_2\hat{\alpha}_2'\hat{\beta}_2' \sum_{ijk\,pqr} \sum_{\gamma_2} \langle ip;\, \gamma_2|V_{\rm pn}|jq;\, \gamma_2\rangle \times C(\alpha_2\beta_2,\, \alpha_2'\beta_2',\, ijk,\, pqr,\, \gamma_2,\, \alpha_4) \times Y(ki;\, \alpha_2)\, Y(kj;\, \alpha_2')\, Y(rp;\, \beta_2)\, Y(rq;\, \beta_2')\,, \quad (4b)$$

$$C(\alpha_2\beta_2,\, \alpha_2'\beta_2',\, ijk,\, pqr,\, \gamma_2,\, \alpha_4) = \sum_l (2l+1) \begin{Bmatrix} \alpha_2 & \beta_2 & \alpha_4 \\ l & k & i \end{Bmatrix} \times \begin{Bmatrix} \alpha_2' & \beta_2' & \alpha_4 \\ l & k & j \end{Bmatrix} \begin{Bmatrix} p & i & \gamma_2 \\ l & r & \beta_2 \end{Bmatrix} \begin{Bmatrix} q & j & \gamma_2 \\ l & r & \beta_2' \end{Bmatrix}. \quad (4c)$$

In eq. (4) ijk (pqr) label proton (neutron) single-particle states while

$$Y(ij;\alpha_2)=\sqrt{1+\delta_{ij}}\,X(ij;\alpha_2)\,,$$

and $V_{\rm pn}$ is the proton-neutron interaction. Throughout this paper we use the same symbols to denote a state as well as the corresponding angular momentum as seen e.g. in the $6j$ symbols of eq. (4c). Eq. (4) corresponds to the MSM diagram shown in fig. 1.

In r-representation the α-particle wave function is

$$\psi(\boldsymbol{r}_1\boldsymbol{r}_2\boldsymbol{r}_3\boldsymbol{r}_4;\alpha_4)=\sum_{\alpha_2\beta_2}X(\alpha_2\beta_2;\alpha_4)(\psi(\boldsymbol{r}_1\boldsymbol{r}_2;\alpha_2)\psi(\boldsymbol{r}_3\boldsymbol{r}_4;\beta_2))_{\alpha_4}\times\psi(\text{core})\,, \qquad (5)$$

where $\boldsymbol{r}_1$, $\boldsymbol{r}_2(\boldsymbol{r}_3,\boldsymbol{r}_4)$ are the coordinates of the two protons (neutrons) in the correlated two-particle wave function. Since in the free α-particle the two neutrons and the two protons are both in a singlet state, in the two-particle wave function only the singlet component should be considered, e.g. for protons it is

$$\psi(\alpha_2;\boldsymbol{r}_1\boldsymbol{r}_2)=(\chi_1\chi_2)_0\sum_{p\leqslant q}X(pq;\alpha_2)\hat{j}_p\hat{j}_q[C(pq;\boldsymbol{r}_1\boldsymbol{r}_2)-(-1)^{p+q-\lambda}C(qp;\boldsymbol{r}_1\boldsymbol{r}_2)]\,, \qquad (6a)$$

where, with standard notation,

$$C(pq;\boldsymbol{r}_1\boldsymbol{r}_2)=R_p(r)R_q(r)(-1)^{l_p+1/2+j_p+\lambda}\begin{Bmatrix} l_p & j_p & \frac{1}{2}\\ j_q & l_q & \lambda\end{Bmatrix}\sum_{m_p,m_q}\langle l_pm_pl_qm_q|\lambda\mu\rangle Y_{l_pm_q}(\Omega_2)\,. \qquad (6b)$$

and a similar expression for neutrons.

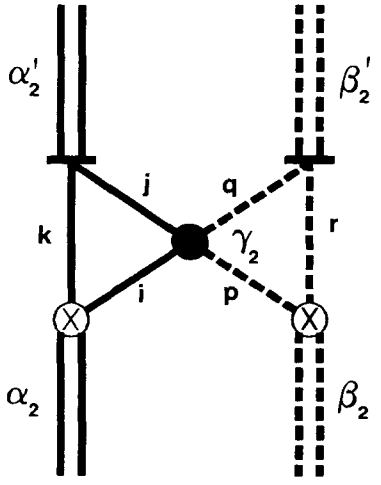

Fig. 1. MSM diagram [15]) corresponding to the interaction between two basis elements in eq. (3). Full (dashed) lines correspond to proton (neutron) excitations. The crossed circle vertices are the wave functions amplitudes, e.g. $X(ki;\alpha_2)$, the full dot is the interaction and the one-bar vertices are the projections of the composed state upon the corresponding components, e.g. $\langle\alpha_2'|(c_k^+c_i^+)_{\alpha_2'}|0\rangle/\sqrt{1+\delta_{ik}}$. In this diagram the one-bar and the crossed circle vertices coincide with each other (except for a complex conjugate operation, which is irrelevant for real interactions).

For the ground state of a system with two identical particles outside a closed-shell core, eq. (6) shows strong clustering features if enough configurations are included in the two-particle representation [11]). This is a property induced by the pairing interaction. If only one configuration is included the absolute value of the wave function (6) becomes symmetrical in the angle θ between the vectors $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ with respect to $\theta = \frac{1}{2}\pi$ (see fig. 2) [ref. [12])]. The physical reason for this is that the inclusion of only one configuration is equivalent to neglecting the interaction. In other words, the amplitudes X are a measure of the strength of the pairing correlation. In the same way, the amplitudes $X(\alpha_2\beta_2;\, \alpha_4)$ in eq. (3) are a measure of the neutron-proton interaction. If this interaction is neglected the states α_4 would simply be the set of MSM basis states. For instance, the ground state of ^{212}Po would be

$$|^{212}\mathrm{Po(g.s.)}\rangle = |^{210}\mathrm{Pb(g.s.)} \otimes {}^{210}\mathrm{Po(g.s.)}\rangle\,. \tag{7}$$

This description of the state ^{212}Po(g.s.) was used in the past in a number of well-known calculations [7,16]). Although in this description the neutron-proton interaction is neglected, the simplicity of eq. (7) made it possible to analyze the relation between α-decay and α-clustering in detail and a good understanding of the α-decay process was obtained. But a proper (exact) treatment of *all* interactions with the shell model (TDA) requires the use of the complete basis in eq. (3).

The clustering of the four nucleons which eventually form the α-particle would be produced by the interplay among the different terms in eq. (3). The neutron-neutron and the proton-proton clustering should appear through the wave functions labelled by α_2 and β_2. That is, one expects that all relevant correlated states entering eq. (3) show clustering features similar to those appearing in the ground state of two-particle systems [11-13,16-18]). The neutron-proton clustering should be induced by a coherent contribution to the wave function $|\alpha_4\rangle$ of all components. That is, the signs of the wave function amplitudes $X(\alpha_2, \beta_2, \alpha_4)$ should be such that the 12-dimensional function (5) strongly increases for vanishing values of the relative distances among neutrons and protons.

If the description of α-clustering is valid in a region beyond the nuclear surface, the Pauli principle among the particles in the core and those in the α-particle is not effective and one can apply the two step mechanism proposed by Gamow [7]). The alpha-decay width is then

$$\Gamma_{\mathrm{L}}(R) = 2\gamma_{\mathrm{L}}^2(R) P_{\mathrm{L}}(R) \tag{8}$$

where the reduced width $\gamma_{\mathrm{L}}(R)$ is related to the formation amplitude $F_{\mathrm{L}}(R)$ at the point R by

$$\gamma_{\mathrm{L}}(R) = \sqrt{\frac{\hbar^2 R}{2\mu}}\, F_{\mathrm{L}}(R)\,. \tag{9a}$$

The formation amplitude is

$$F_L(R) = \int (\varphi_d \varphi_\alpha Y_L)^* \varphi_m \, dr, \tag{9b}$$

where φ_d and φ_m are the daughter- and mother-nucleus wave functions, respectively, and φ_α is the intrinsic α-particle radial wave function for which we use a standard form [7]). The coordinate R is the distance between the outgoing α-particle and the centre of the daughter nucleus. Note that in the case of the decay of ^{212}Po the daughter nucleus is the core, i.e. the vacuum of excitations, so that in this special case eq. (9b) becomes

$$F_L(R) = \sum_{\alpha_2 \beta_2} X(\alpha_2 \beta_2; \alpha_4) \int d\hat{R} \, d\xi_\alpha \, (\varphi_\alpha(\xi_\alpha) Y_L(\hat{R}))^*_{\alpha_4} (\psi(r_1 r_2; \alpha_2) \psi(r_3 r_4; \beta_2))_{\alpha_4}, \tag{10}$$

where ξ_α are the 9 coordinates corresponding to the 3 relative distance vectors defining the α-particle.

Finally, in eq. (8) $P(R)$ is the penetration factor through the Coulomb barrier at the point R from the centre of the daughter nucleus, i.e.

$$P_L(R) = kR/(G_L^2(R) + F_L^2(R)), \tag{11}$$

where $G_L(R)$ and $F_L(R)$ are the irregular and regular Coulomb functions, respectively.

The decay width calculated by using eq. (8) may depend strongly upon the distance R since the penetration factor (11) is very dependent upon R. In this case, the calculation would be meaningless since small changes in the distance R would be enough to overrun all other details of the formalism and a fitting of experimental data would always be possible. On the other hand, if our treatment of the α-clustering is adequate the decay width which is calculated by means of eq. (8) should, in principle, be independent of R in a region around the nuclear surface, e.g. where the α-particle is alredy formed. Therefore a requirement which should be fullfilled by our calculation is that Γ_L is only weakly dependent on R in that region. In the next section we will use this as a criterion to check our calculations.

The calculation of the 11-dimension integral in eq. (10) may present a numerical problem. To avoid this we use a harmonic-oscillator representation so that the integrals can be performed analytically. The use of such representations have successfully been used in alpha decay [7]) as well as in other processes where the continuum plays a role as, e.g. in the study of the building up and decay of giant resonances [19]). Moreover, in ref. [5]) it was used as representation the set of single-particle states solutions of a Woods–Saxon potential and the integrals were performed numerically. We have checked that the results of ref. [5]) are also obtained within the harmonic-oscillator basis that we will use in the next section.

3. Applications

In this section we will consider the α-decay ^{212}Po(g.s.) $\rightarrow \alpha + {}^{208}$Pd(g.s.) which is a convenient case to analyze, within our formalism, the α-clustering features in heavy spherical nuclei. As discussed in the previous section, the MSM representation that we will use consists of the set of states $\{^{210}\text{Po}(\alpha_2) \otimes {}^{210}\text{Pb}(\beta_2)\}$. The amplitudes $X(\alpha_2\beta_2; \alpha_4)$ (see eq. (3)) and the four-particle energies $W(\alpha 4)$ are calculated by means of eq. (4). But as the first step of the MSM we have to calculate the two-particle states. These we took from the calculation of ref. [20]) with the modifications discussed in ref. [21]), so that most of the available two-particle experimental data are well described by our two-particle interaction. Our single-particle representation consisted of harmonic-oscillator states up to the major shell $N = 15$ (14) for neutrons (protons). For the single-particle states which are above the first major shells of ref. [20]) (i.e. $N = 6$ (5) for neutrons (protons)) we used a surface delta interaction. With the two-particle energies and wave functions thus evaluated, in the second step we calculated the MSM matrix elements of eq. (4) and the corresponding four-particle energies and wave functions. For the state ^{212}Po(g.s.) the calculated energy agrees with the corresponding experimental value within 100 keV. The corresponding wave function is given in table 1. As one would have expected, the most important two-particle states in table 1 are the yrast states. Therefore, the analysis of the clustering of the four nucleons outside the ^{208}Pb core requires an understanding of the behaviour of the two-particle yrast wave functions. One knows that for the ground state the two particles are strongly clustered on the nuclear surface if a large enough number of configurations is included in the calculation[5,11-13,17,18]). The physical reason behind this is that the continuum, which plays an important role in inducing the clustering, is described, albeit approximately, by the high-lying configurations. As a typical example of the influence of the continuum upon two-particle clustering we show in fig. 2 the wave function of ^{210}Pb(g.s.) calculated on the nuclear surface according to eq. (6). The strong maximum that is built up at $\theta = 0$ as the number of configurations is increased, is

TABLE 1

Main components of the ^{212}Po(g.s.) wave function in terms of the MSM basis $\{^{210}\text{Po}(\alpha_2) \otimes {}^{210}\text{Pb}(\beta_2)\}$

^{210}Po	^{210}Pb	Amplitude
0_1^+	0_1^+	0.932
2_1^+	2_1^+	−0.259
4_1^+	4_1^+	0.062
6_1^+	6_1^+	0.028
8_1^+	8_1^+	−0.012

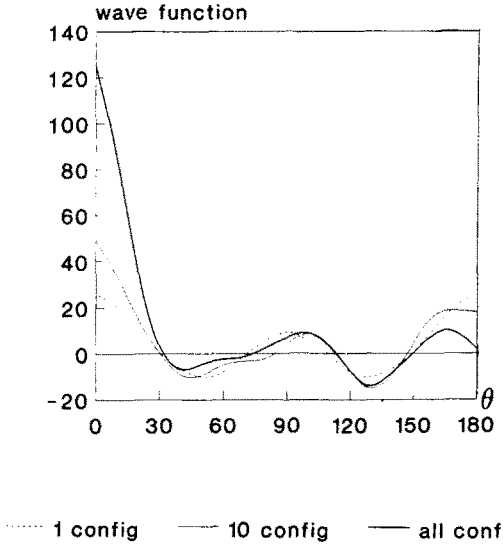

Fig. 2. Two-particle wave function (in units 10^{-5} fm^{-3}) corresponding to the state ^{210}Pb(g.s.). The neutron coordinates are $r_1 = r_2 = 8$ fm. The angle θ is the angle between the vectors $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$. The number of configurations included in the calculation is indicated in the figure.

a result of the interplay among configurations carrying even and odd orbital angular momenta[12]). This characteristic feature of the pairing interaction is related to the enhancement of two-particle cross sections leading to pairing vibrations [13]).

The clustering among identical particles induced by the monopole components of the wave function in table 1 is responsible for most of the contribution to the calculated alpha-decay width [5,7,22]). Yet, if only this component is included in the calculation, the resulting width is too small compared with the corresponding experimental data [7,16]). It is natural to assume that the reason for this shortcoming is the lack of the neutron-proton interaction, which is implied by neglecting the two-particle multipole states. In other words, the components of multipolarities other than monopole in table 1 are a result of the neutron-proton interaction and, therefore, may induce the neutron-proton interaction and, therefore, may induce the neutron-proton clustering missing in the monopole component. In order to probe this assumption we shall first analyze the degree of "clusterization" of the multipole two-particle states.

The most convenient way of analyzing the clustering of multipole pairing states is to study their building up, as in fig. 2. We have now an additional degree of freedom, namely the $(2\lambda + 1)$ projections μ of the angular momentum in eq. (6). Classically one expects that only $\mu = 0$ would show clustering features. Otherwise the motions of the two neutrons would occur on different planes [23]). Since we have spherical symmetry in our decay process, there is not any μ-dependence in the calculated width. It is therefore enough for us to analyze the case $\mu = 0$ only.

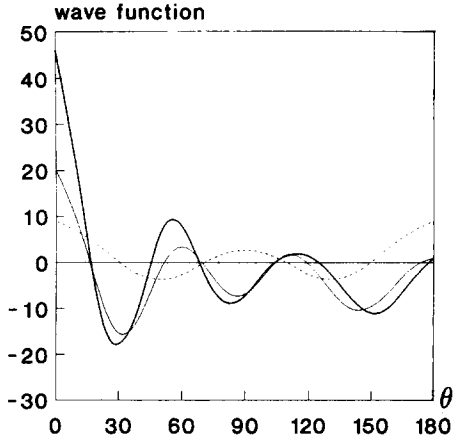

Fig. 3. As fig. 2 for the state ^{210}Pb(2_1^+).

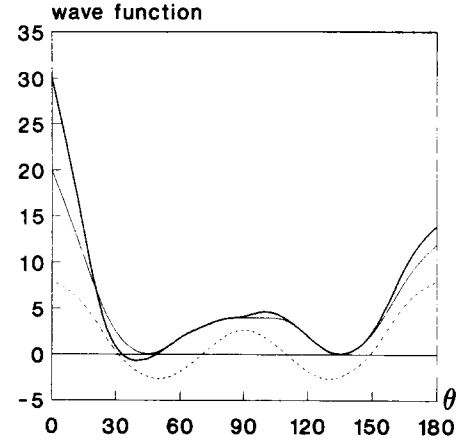

Fig. 4. As fig. 2 for the state ^{210}Pb(6_1^+).

In fig. 3 we present the wave function ^{210}Pb(2_1^+) on the nuclear surface as a function of the angle θ between the position vectors of the two neutrons and the number of configurations. By comparing with fig. 2 one sees that there is also here a two-particle clustering, but it is much weaker than in the case of monopole states. This weak clustering is about the same for all other yrast states. As an illustration we show in fig. 4 the case ^{210}Pb(6_1^+). Yet, the weak clustering among identical particles moving in multipole states may not present a problem for our α-decay calculation because the role of these states would be to induce the clustering of the non-identical particles. A necessary condition for this is that all configurations in table 1 contribute coherently in building up the mother wave function at the centre of the cluster. That is, all components in the sum of eq. (10) should have the same sign when all coordinates $\boldsymbol{r}_i$ ($i = 1, \ldots, 4$) coincide in a point on the nuclear surface. We have checked that indeed this is the case, although again here the effect is not very dramatic. To show this enhancement as well as the clustering of the four-particle wave function, one may proceed as in the two-particle case and show different figures corresponding to the different coordinates in eq. (10). However, the number of relevant coordinates is now very large and since the neutron-proton clustering is not very impressive there is not any particlar choice of two coordinates which will clearly show that clustering. Another way of looking upon this is by analyzing the formation amplitude (9b) directly. In this case there is only one coordinate, namely the relative distance R between the centre of the mother nucleus and the α-particle.

In fig. 5 we present the partial contributions to the formation amplitude F_L from the two most important configurations in table 1 including only the signs of the

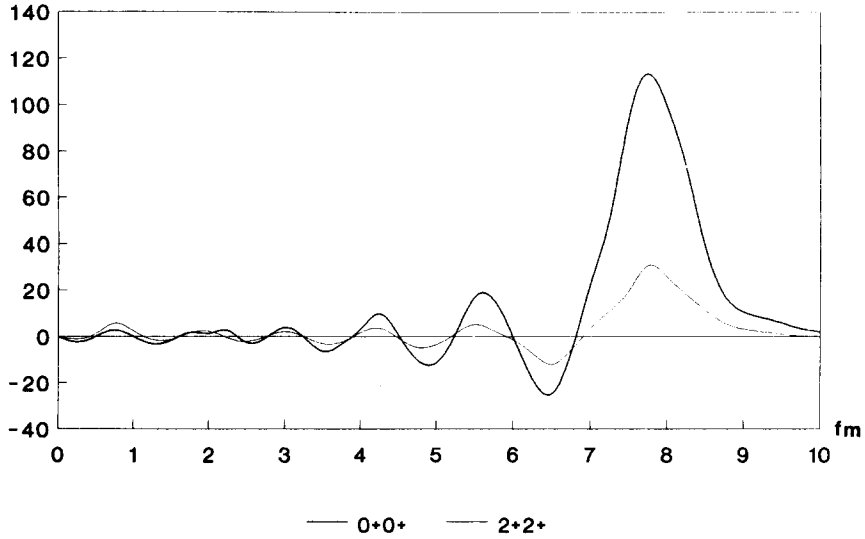

Fig. 5. Formation amplitude (in units $fm^{-3/2}$) corresponding to the partial contributions of the configurations $\{^{210}Po(0_1^+)\otimes^{210}Pb(0_1^+)\}$ and $\{^{210}Po(2_1^+)\otimes^{210}Pb(2_1^+)\}$ in eq. (10).

wave function amplitudes, i.e. those formation amplitudes correspond to "pure" configurations. The other configurations follow the same pattern as the quadrupole one. In particular, for all configurations the formation amplitude has its maximum on the nuclear surface, where the alpha particle is formed [5,7]). Moreover, in this region all configurations contribute in phase increasing the formation amplitude, eq. (10). That is, the α-clustering of the mother nucleus wave function increases as the different multipole channels are included. This is an illustration of the role of the neutron-proton alignment. As discussed in the previous section, the influence of the neutron-proton interaction is manifested through the mixing of configurations in the mother nucleus wave function. If only the monopole channel is included, as in previous calculations [4-7]), the neutron-proton interaction does not play any role from the point of view of clustering. In our case the monopole channel is very dominant and the effect of the other channels is small. The most important of the channels other than those in fig. 5 is the hexadecapolar, which contributes nearly an order of magnitude less than the quadrupole channel. Since the functions F_L in fig. 5 have still to be multiplied by the absolute values of the wave function amplitudes, the dominance of the monopole channel is even larger than appears in fig. 5. Therefore the enhancement of the total formation amplitude with respect to the "pure" monopole-channel case considered before [4-7]) is negligible and the neutron-proton clustering is unfortunately small.

Our calculated decay width presents very similar features to the one calculated

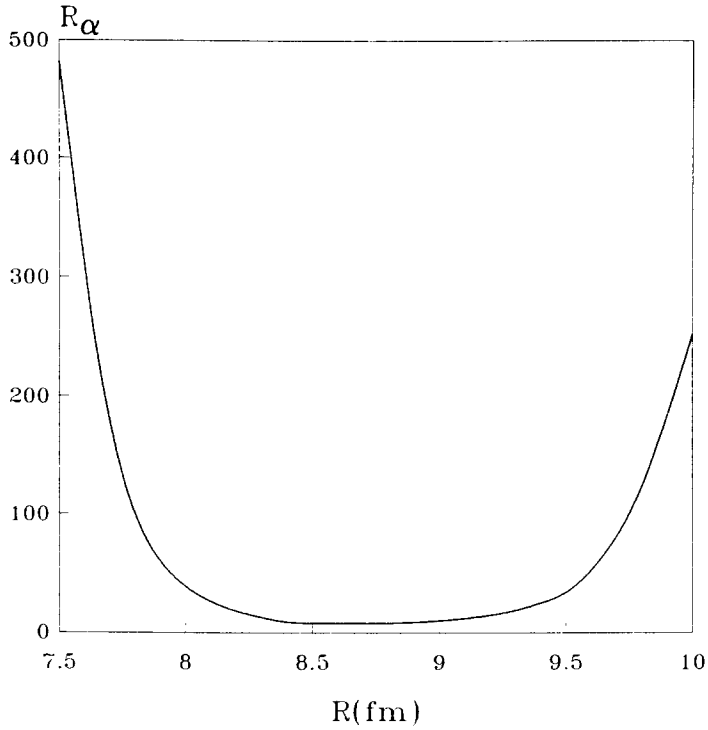

Fig. 6. Ratio $R_\alpha = \Gamma_{\text{exp}}(R)/\Gamma_{\text{theory}}(R)$ as a function of the distance R between the alpha particle and the daughter nucleus.

upon the distance R in a region around the nuclear surface [16]), as seen in fig. 6. Although the more the clustering features are pronounced in the initial state, the better the assumptions of the Gamow theory are fulfilled, fig. 6 indicates that the Gamow separability of the α-decay process is still valid in our case. Yet, the calculated decay width on the nuclear surface is about one order of magnitude smaller than the corresponding experimental value. It thus seems that our way of treating the neutron-proton interaction does not account for the clustering of the non identical nucleons in the alpha particle. But we just applied in this paper the shell-model prescription to calculate a nucleus with two neutrons and two protons outside a frozen core [24]). In view of this we do not have a definite explanation of the reason why the calculated alpha-decay width is too small, except that our treatment of the continuum may still be defective. One may also argue that the Pauli principle among the nucleons in the core and those in the alpha particle is defective in our analysis [6]). However, it was shown in ref. [7]) that this is not the case. More likely is that collective particle-hole correlations (core polarization), which we did not include in our treatment, plays a role in alpha-particle decay from spherical nuclei, as suggested long ago [25]).

4. Summary and conclusions

In this paper we have studied the alpha clustering and corresponding alpha decay of a spherical normal nucleus consisting of two neutrons and two protons outside a frozen core. We applied the formalism to the alpha-decay process in the nucleus ^{212}Po. Previous calculations had shown that including only the monopole (pairing) interaction among identical particles and within a large single-particle representation the two neutrons and the two protons which eventually become the alpha particle are strongly clustered on the nuclear surface [11]). The corresponding calculated decay width is also strongly enhanced in such calculations [7,16]) but its value is still smaller than the corresponding experimental value by about one order of magnitude. It thus seemed natural to assign that shortcoming to the inadequate treatment of the two body interaction. In this paper we have attempted to remedy this by including also the neutron-proton interaction within the framework of the multistep shell-model method [15]). In our case the MSM basis consisted of the tensorial product of the two-particle correlated states, i.e. $\{^{210}\mathrm{Po}(\alpha_2)\otimes^{210}\mathrm{Pb}(\beta_2)\}$. The influence of the continuum upon the formation of the alpha particle was considered by using a large (harmonic oscillator) single-particle representation, as it was the case in previous calculations as well as in other processes where the continuum plays an important role (e.g. in the analysis of the formation and decay of giant resonances). The importance of the neutron-proton interaction in our formalism is reflected in the degree of mixing in the MSM basis describing the mother nucleus wave function, i.e. ^{212}Po(g.s.). But, as seen in table 1, that wave function consists mainly of the configuration $\{^{210}\mathrm{Po(g.s.)}\otimes^{210}\mathrm{Pb(g.s.)}\}$, as expected [15,23]). Although this may indicate that the influence of the interaction among non-identical particles is weak, one has to consider that the effect that one is looking for is not very large. Indeed, the inclusion of only the monopole interaction increases the value of the calculated decay width by about five orders of magnitude [7,16]) and thus the additional order of magnitude which is still required seems to be a relatively small percentage of the total value.

We studied the clustering features of the two-particle states which are dominant in the description of ^{212}Po(g.s.) shown in table 1. We found that the strong clustering induced by the pairing interaction upon identical nucleons moving in the ground states (pairing vibrations) is not very impressive for the multipole states. Yet, this does not necessarily mean that there is not clustering among the neutrons *and* protons which form the alpha particle. To check this point we analyzed the formation amplitude of the alpha particle for each MSM configuration separately. We found that all configurations contribute with the same phase on the nuclear surface, thus increasing the value of the decay width. However, the contribution of the multipole states is very weak and they just compensate for the smaller contribution of the monopole states due to the MSM configuration mixing. The final result is that the calculated decay width does not differ very much from the one in which only the

monopole interaction was considered. The positive features of those calculation are also present here. In particular, our decay width is weakly dependent of the distance between the alpha particle and the mother nucleus in a region around the nuclear surface [5,16]). This is a test of the reliability of our approximations. But our decay width is also too small by about one order of magnitude. A reason for this shortcoming may be that we did not include core polarization degrees of freedom in our mother nucleus wave function [25]). Another possible reason for the discrepancy between our calculations and the corresponding experimental data is that in the analysis of absolute alpha-decay width a proper treatment of the continuum, including the possibility of single-particle decay, is required. Such a treatment is necessary in the analysis of particle decay of giant resonances [26,27]) and it might also be necessary here.

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